You will each adopt a group and apply to it the ideas we learn during the semester, as well as researching other information relevant to your group (historical context, applications, interpretations, for example). Your work will culminate in a paper discussing your results. There will also be a couple short papers and informal discussions in class along the way. My hope with this assignment is that having a specific group to investigate whenever we learn new concepts and properties will help you to grasp the concepts more deeply. Furthermore, I hope you become more comfortable with math research – figuring out what questions you need to ask, and then answering them.

HOW THIS PROJECT WILL BE GRADED:
Your final grade on this project will be based upon your grade on the two interim short papers, whether you met with me to discuss your project at least three times (and were adequately prepared-see the schedule below for a list of what you should have investigated for each meeting), your participation in the class discussions of your groups, and of course mainly on "the big paper".

CONCEPTS TO INVESTIGATE:
I am including below a list of algebraic properties and concepts for you to investigate. (Of course, you'll learn the vocabulary as we go along.) Each adopted group will vary as to which properties are easy to investigate and which are harder; you may not be able to develop information about some of the concepts, but you may be able to find information not referred to on this list. Please don't assume any of the items on this list are irrelevant to your group until you discuss them with me.

- Verify your example **is** a group. This should of course be preceded by whatever definitions and explanations of the elements and operation are necessary. In some cases, the elements of the group will be fairly self-evident while in others, this discussion will be crucial and somewhat lengthy.
- Is your group Abelian?
- Find the order of the group
- Find the order of each element in the group
- Interpretations(s), applications of the group
- Is your group cyclic?
- Can you find generators of your group? (This should be addressed whether or not the group is cyclic - groups can be generated by two, three, or more elements.)
- Find the center of your group.
- Find the centralizer of each element. (This may or may not be possible for infinite groups; if it's not possible, investigate as far as you can and draw whatever conclusions are possible.)

- Subgroups: if possible (and sensible) give a complete subgroup lattice or a description of all subgroups; if neither of these is practical, then give and discuss an interesting collection of subgroups.
- Isomorphisms, automorphisms, and inner automorphisms. If practical, investigate the automorphism group and the inner automorphism group.
- Normal subgroups
- Quotient (factor) groups

WRITING THE BIG PAPER:
In your culminating paper, you should include all of the information you find on the above properties and concepts, as well as anything else you learn about your group along the way. Your paper should not be a collection of facts, or even a list of results; I'm sure you want to interest and inform us, so make your paper clear and engaging – mathematical writing doesn't have to put the audience to sleep or mentally exhaust the audience–assume your audience consists of your classmates. Writing about math in a clear, straightforward, and even eloquent way **is** possible, but I know it is difficult. I am happy to help you out, so come to me as often as you need to, and take advantage of your classmates' good nature also.

Your culminating paper should include a title page and an abstract. You should consult other sources (at least one is required) for information not in our textbook. These sources should of course be properly referred to in the body of the text and in a bibliography. Expect your paper to be a minimum of 5-7 pages; it may well end up being 10 pages or more.

GRADING THE BIG PAPER:

The criteria I'll use to grade your paper can be summarized as follows:

- **Content: correctness, completeness, and depth:**
  The point of your paper, of course, is the presentation of the results of your investigations. These include finding and proving or demonstrating all the basic properties of your group, determining additional properties which only your group may have, and possibly finding and presenting historical contexts, applications, or interpretations of your group. Your findings should be correct, you should discover everything you reasonably can about your group, and you should investigate in whatever depth seems called for in each particular case.
- **Clarity:**
  While the mathematical results are the point of the paper, there's no purpose to writing a paper if the results are not presented clearly. Remember, it is not the reader's job to understand what you are writing, it is your job to make yourself easily understood. To be understood, your writing must be clear on two levels.

  On the "global" level, you should pay attention to how the whole paper flows. Ask yourself in which order you should present the concepts–should you discuss whether

the group is cyclic before or after you discuss whether the group is Abelian? Should you give an example before or after you give a proof? Make sure you are consistent in your notation and in your treatment of the subject. Are your arguments convincing?

On the "local" level, you should pay attention to each paragraph. You should strive for clarity in each phrase, sentence, argument, figure, table, etc. Are the ideas in a paragraph all related, and are they in the order that best conveys your point? Is it clear to the reader where each logical step comes from? When you change trains of thought, do you begin a new paragraph? Are the figures and tables referred to at the appropriate time, and are they easily found by the reader?

- **Grammar, Punctuation, and Neatness:**
  Finally, while clarity is key to how much a person enjoys reading your paper, it is also important that they believe your results, and for that matter, even pay attention while they are reading. Clarity is an important factor in these issues, but so too is neatness, punctuation, and grammar. A paper that is visually a mess does not inspire confidence in the author, and neither do errors in grammar or punctuation. Worse even than dispelling confidence in your results, though, messiness, poor punctuation and grammar can be confusing and even misleading. Make sure the paper is neat and grammatically correct, and people will trust that you have put the same care into your results.

Note: For any given property that I've asked you each to investigate, the difficulty of making arguments or finding additional information will vary from adopted group to adopted group. Because I'm aware of this, I may well consider a paper with fewer but more difficult to ascertain facts to be worth as much or more as a paper with more but easy to ascertain facts. If you feel that many of the facts in your paper were easy to ascertain (for instance, if your group is Abelian and of a given order), you may want to investigate other aspects of your group in more depth than I would expect from a student whose group posed more difficulties. So when it comes time to select your group, don't fret that some groups are easier than others.

TIME SCHEDULE:

September 6	Adopt groups
September 15	Short paper due, verifying your example **is** a group. This paper should define and explain the elements and the operation of your group, and should make sure the operation is well-defined. You should also discuss whether your group is Abelian. Be prepared to tell the class a bit about your group – what is it that defines your group and makes it interesting or different?
September 15-September 29	Research and develop information about your group. During this time, you are required to schedule at least one meeting with me to discuss your progress. Bring **all** your notes and references whenever you come to see me.
September 29	By this date, you should have finished investigating the order of your group and of each element, the center and centralizers, cyclic subgroups, and made good strides on discussing all subgroups of your group. If you have a finite number of subgroups, you should have a subgroup lattice.
September 30-October 18	Continue researching your group. During this period, you are again required to schedule at least one meeting with me to discuss your progress. Bring all notes and references.
October 20	Short paper due, explaining something interesting and something confusing about your group. Again, be prepared to discuss what's interesting and confusing. By this time, you should have investigated generators of your group, isomorphisms between your group and others, automorphisms, inner automorphisms, and have begun thinking about cosets. Also be investigating historical context, applications, and interpretations.
October 21-November 10	Continue researching your group. Again, you should meet with me, bringing your references. During this period, you should investigate normal subgroups and factor groups. Continue investigating historical context, applications, and interpretations. You should also be refining all the previous ideas,and be writing your first draft.
November 13	First draft of paper due. Peer review takes place.
November 14-November 20	Respond to suggestions, refine your paper.
November 20	Second draft of paper due, this time to me.
November 27-December 4	Respond to my suggestions, and refine your paper still more.
December 4	Final draft of the paper due. Be prepared to discuss your group with the class.

ADOPTABLE GROUPS

D_6	The Dihedral Group of order 12
$\mathbb{C}_{10}$	The 10 roots of unity with complex number multiplication
Q_4	The Quaternion group
$\mathbb{Z}_2[x]$	The additive group of polynomials with coefficients of either 0 or 1
T	The symmetries of a regular tetrahedron
M	The multiplicative group of 2×2 matrices of the form $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ where x is a real number.
$U(24)$	The set of integers less than and relatively prime to 24 under multiplication modulo 24
$GL(2, \mathbb{Z}_2)$	The multiplicative group of invertible 2×2 matrices with entries in $\mathbb{Z}_2$.
$SL(2, \mathbb{R})$	The multiplicative group of 2×2 matrices with real entries and with determinant 1
S_5	The set of all permutations of $\{1, 2, 3, 4, 5\}$
—	All groups of order 9
—	All groups of order 8

This assignment was suggested to me by Professor Annalisa Crannell at Franklin and Marshall, who in turn adapted it from an assignment described by Ralph Czerwinski, "A Writing Assignment in Abstract Algebra", *PRIMUS* **4** no. 2, 117-124 (1994).