You will each adopt a group and apply to it the ideas we learn during the semester, as well as researching other information relevant to your group (historical context, applications, interpretations, for example). Your work will culminate in a paper discussing your results. There will also be a couple short papers and informal discussions in class along the way. In my experience, having a specific group to investigate whenever we learn new concepts and properties will help you to grasp the concepts more deeply. Furthermore, I hope you become more comfortable with math research – figuring out what questions you need to ask, and then answering them.

HOW THIS PROJECT WILL BE GRADED:
Your final grade on this project will be based mainly upon your grade the culminating paper, but also partially on two interim short papers, as well as whether you met with me to discuss your project at least two times (and were adequately prepared-see the schedule below for a list of what you should have investigated for each meeting) and your participation in the class discussions of your groups.

CONCEPTS TO INVESTIGATE:
I am including below a list of algebraic properties and concepts for you to investigate. (Of course, you'll learn the vocabulary as we go along.) Each adopted group will vary as to which properties are easy to investigate and which are harder; you may not be able to develop information about some of the concepts, but you may be able to find information not referred to on this list. Please don't assume any of the items on this list are irrelevant to your group until you discuss them with me.

- Verify your example **is** a group. This should of course be preceded by whatever definitions and explanations of the elements and operation are necessary. In some cases, the elements of the group will be fairly self-evident while in others, this discussion will be crucial and somewhat lengthy.
- Is your group Abelian?
- Find the order of the group
- Find the order of each element in the group
- Interpretations(s), applications of the group
- Is your group cyclic?
- Can you find generators of your group? (This should be addressed whether or not the group is cyclic - groups can be generated by two, three, or more elements.)
- Find the center of your group.
- Find the centralizer of each element. (This may or may not be possible for infinite groups; if it's not possible, investigate as far as you can and draw whatever conclusions are possible.)

- Subgroups: if possible (and sensible) give a complete subgroup lattice or a description of all subgroups; if neither of these is practical, then give and discuss an interesting collection of subgroups.
- Isomorphisms, automorphisms, and inner automorphisms. If practical, investigate the automorphism group and the inner automorphism group.
- Normal subgroups
- Quotient (factor) groups

WRITING THE BIG PAPER:
In your culminating paper, you should include all of the information you find on the above properties and concepts, as well as anything else you learn about your group along the way. Aim the paper at your classmates – people who know what you do about groups in general, but have given your particular group no thought. As I'm sure you want to interest and inform us, your paper should not be a collection of facts, or even a list of results. Make your paper clear and engaging – mathematical writing doesn't have to put the audience to sleep or mentally exhaust the audience. Writing about math in a clear, straightforward, and even eloquent way **is** possible, but I know it is difficult. I am happy to help you out, so come to me as often as you need to, and take advantage of your classmates' good nature also.

Your culminating paper should include a title page and an abstract. You should consult other sources (at least one is required) for information not in our textbook. These sources should of course be properly referred to in the body of the text and in a bibliography. Expect your paper to be a minimum of 7 pages; it may well end up being 10 pages or more.

You will each get the opportunity to read several of your classmates' final drafts.

GRADING THE BIG PAPER:

The criteria I'll use to grade your paper can be summarized as follows:

- **Content: correctness, completeness, and depth:**
  The point of your paper, of course, is the presentation of the results of your investigations. These include finding and proving or demonstrating all the basic properties of your group, determining additional properties which only your group may have, and possibly finding and presenting historical contexts, applications, or interpretations of your group. Your findings should be correct, you should discover everything you reasonably can about your group, and you should investigate in whatever depth seems called for in each particular case.
- **Clarity:**
  While the mathematical results are the point of the paper, there's no purpose to writing a paper if the results are not presented clearly. Remember, it is not the reader's job to understand what you are writing, it is your job to make yourself easily understood. To be understood, your writing must be clear on two levels.

On the "global" level, you should pay attention to how the whole paper flows. Ask yourself in which order you should present the concepts–should you discuss whether the group is cyclic before or after you discuss whether the group is Abelian? Should you give an example before or after you give a proof? Make sure you are consistent in your notation and in your treatment of the subject. Are your arguments convincing?

On the "local" level, you should pay attention to each paragraph. You should strive for clarity in each phrase, sentence, argument, figure, table, etc. Are the ideas in a paragraph all related, and are they in the order that best conveys your point? Is it clear to the reader where each logical step comes from? When you change trains of thought, do you begin a new paragraph? Are the figures and tables referred to at the appropriate time, and are they easily found by the reader?

- **Grammar, Punctuation, and Neatness:**
  Finally, while clarity is key to how much a person enjoys reading your paper, it is also important that they believe your results, and for that matter, even pay attention while they are reading. Clarity is an important factor in these issues, but so too is neatness, punctuation, and grammar. A paper that is visually a mess does not inspire confidence in the author, and neither do errors in grammar or punctuation. Worse even than destroying our faith in your results, though, messiness, poor punctuation and grammar can be confusing and even misleading. Make sure the paper is neat and grammatically correct, and people will trust that you have put the same care into your results.

Note: For any given property that I've asked you each to investigate, the difficulty of making arguments or finding additional information will vary from adopted group to adopted group. Because I'm aware of this, I may well consider a paper with fewer but more-difficult-to-ascertain facts to be worth as much or more as a paper with more but easy-to-ascertain facts. If you feel that many of the facts in your paper were easy to ascertain (for instance, if your group is Abelian and of a given order), you may want to investigate other aspects of your group in more depth than I would expect from a student whose group posed more difficulties. So you and your classmates start comparing your work, don't fret that some groups are easier than others.

This assignment was suggested to me by Professor Annalisa Crannell at Franklin and Marshall, who in turn adapted it from an assignment described by Ralph Czerwinski, "A Writing Assignment in Abstract Algebra", *PRIMUS* **4** no. 2, 117-124 (1994).

TIME SCHEDULE:

9/8	**Adopt groups**
9/17	**Short paper due**, verifying your example **is** a group. This paper should define and explain the elements and the operation of your group, and should make sure the operation is well-defined. You should also discuss whether your group is Abelian. **Be prepared to tell the class a bit about your group** – what is it that defines your group and makes it interesting or different?
9/20-10/08	During this time, you are required to **schedule at least one meeting with me to discuss your progress.** Bring **all** your notes and references whenever you come to see me. During this period, you should investigate the order of your group and of each element, the center and centralizers, cyclic subgroups, and make good strides on discussing all subgroups of your group. If you have a finite number of subgroups, you should have a subgroup lattice. You should also have found generators of your group, isomorphisms between your group and others, automorphisms, and inner automorphisms. Also be investigating historical context, applications, and interpretations.
10/22	**Short paper due**, discussing at least one interesting and one confusing aspect of your group. Also, honestly let me know how far you've gotten in your investigations. Many people find that it's helpful to type up everything they've learned to date. Even though the exposition will undoubtedly change, it makes the final paper easier to work on. **Be prepared to discuss what's interesting and confusing.**
10/25-11/12	Continue researching your group. Again, **you are required to schedule at least one meeting with me during this time, bringing your references.** Throughout these weeks, you should investigate cosets, normal subgroups and factor groups. Continue investigating historical context, applications, and interpretations. You should also be refining all the previous ideas you developed, **and be writing your first draft.**
11/15	**First draft of paper due.** Peer review takes place.
11/17-11/21	Respond to suggestions, refine your paper.
11/22	**Second draft of paper due**, this time to me.
11/23-12/5	Even before you get your draft back, continue to refine your paper. Once you get your draft back, make any changes I suggest, and continue refining.
12/6	**Final draft due. Be prepared to discuss your group with the class.**
12/8	**Paper Exchange.** You'll receive several of your classmates' papers to read.

ADOPTABLE GROUPS

D_6	The Dihedral Group of order 12
$\mathbb{C}_{10}$	The 10 roots of unity with complex number multiplication
Q_4	The Quaternion group
$\mathbb{Z}_2[x]$	The additive group of polynomials with coefficients of either 0 or 1
$\mathbb{Z}[i]$	The Gaussian Integers – that is, the additive group of complex numbers with integer real and imaginary parts
T	The symmetries of a regular tetrahedron
$SO(2,\mathbb{R})$	The multiplicative group of 2×2 matrices of the form $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ where x is a real number.
H	The (multiplicative) discrete Heisenberg group, $\left\{ \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \right\}$ where $a, b, c \in \mathbb{Z}$
$GL(2,\mathbb{Z}_2)$	The multiplicative group of invertible 2×2 matrices with entries in $\mathbb{Z}_2$.
$SL(2,\mathbb{R})$	The multiplicative group of 2×2 matrices with real entries and with determinant 1
S_5	The set of all permutations of $\{1,2,3,4,5\}$
$\mathcal{T}_2$	The set of translations in $\mathbb{R}_2$, $\{T_{a,b} \mid a, b \in \mathbb{R}\}$, where $T_{a,b} : \mathbb{R}^2 \to \mathbb{R}^2$ by $(x, y) \mapsto (x + a, y + b)$
$\mathcal{F}$	The set of all symmetries of the infinite ornamental pattern in which arrowheads are spaced uniformly a unit apart along a line: $\cdots \to \quad \to \quad \to \quad \to \quad \to \quad \to \cdots$.
—	All groups of order 9 (Example=$\mathbb{C}_9$)
—	All groups of order 8 (Example=$U(24)$)
$\mathbb{Q}_{2,3}$	The set of all rational numbers of the form $2^m 3^n$ with m and n
$U(13)$ & $U(21)$	Investigate and compare the set of integers less than and relatively prime to 13 under multiplication and the set of integers less than and relatively prime to 21 under multiplication