

# Mandelbrot Set (more or less) - The Recursive Process

**Start:** Pick a point  $s = (a, b)$  to be the **seed**.

**Start:** Write  $s$  as a complex number,  $s = a + bi$

**Next:** Let  $z_0 = 0$

1.  $n = 0$
2. Add 1 to  $n$
3. Let  $z_n = z_{n-1}^2 + s$
4. Go to step 2

Form a list of the different values  $z$  takes on, beginning with  $s$ :

$$\{z_0, z_1, z_2, z_3, \dots\}$$

This list is called the **Mandelbrot Sequence** for the seed  $s$ .

- ▶ If the **seed** grows to  $\pm\infty$  (that is, if the terms in the Mandelbrot sequence increase (or decrease, or both, without bound), we say the seed **escapes**.
  - ▶ In that case, the seed is **not** in the Mandelbrot set.
  - ▶ We color the original point in space that corresponds to the seed a color. The color is determined by **how fast** the seed escapes.
- ▶ If the **seed** does not escape, whether its because the terms in the Mandelbrot sequence approach some finite number, or alternate between approaching a couple of finite numbers, then
  - ▶ the seed **is** in the Mandelbrot set
  - ▶ We color the original point in space that corresponds to the seed **black**.

## Examples We've Seen So Far:

- ▶  $s = 1 + 0i$ 
  - ▶ Omitting 0, the Mandelbrot Sequence:  $\{1, 2, 5, 26, 677, \dots\}$
  - ▶ The seed  $(1, 0)$  escapes
  - ▶  $(1, 0)$  is **not** in the Mandelbrot set. Color it some non-black color, based on how fast it escapes
- ▶  $s = 0$ 
  - ▶ Omitting initial 0, the Mandelbrot Sequence:  $\{0, 0, 0, 0, \dots\}$
  - ▶ The seed  $(0, 0)$  is attracted
  - ▶  $(0, 0)$  **is** in the Mandelbrot set. Color it black
- ▶  $s = -0.5 + 0i$ 
  - ▶ Omitting 0, the Mandelbrot Sequence:  
 $\{-0.5, -0.25, -0.4375, -0.3086, -0.4048, -0.3362, \dots\}$
  - ▶ The seed  $(-0.5, 0)$  is attracted
  - ▶  $(-0.5, 0)$  **is** in the Mandelbrot set. Color it black
- ▶  $s = 0 + i$ 
  - ▶ Omitting 0, the Mandelbrot Sequence:  
 $\{i, -1 + i, -i, -1 + i, -i, -1 + i, \dots\}$
  - ▶ The seed  $(0, 1)$  is periodic
  - ▶  $(0, 1)$  **is** in the Mandelbrot set. Color it black