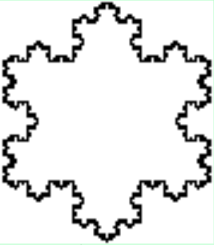
Fractal Dimension



Koch Snowflake = boundary.

Boundaries usually are 1 dimensional.

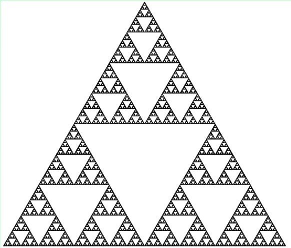
Finite area, infinite boundary.

Lots of boundary squished around area

Does that mean boundary has area?

Is it really 2 dimensional?

Dimension of the Koch Snowflake?



Sierpinski Triangle has Area=0

Is it a boundary, with dimension 1?

Or is it 2 dimensional?

Dimension of the Sierpinski Triangle?

Goal:

To figure out how to find the dimension of a fractal, like the Sierpinski Triangle or the Koch Snowflake

The **defining** property of these fractals has been their self-similarity.

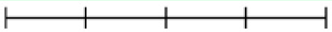
Plan:

To relate dimension and self-similarity

Relating Self-Similarity to Dimension:

Dimension 1:

- Begin with a line segment.
- Divide line segment into equal length pieces.

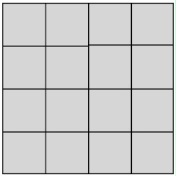


  - Each piece is identical to the original, just smaller.
  - Number of pieces $= N$
    - In this example, $N = 4$
  - Each has length $= 1/N$ of original length.
- To enlarge a piece to be identical to original, multiply length by **scaling factor**
  - Scaling factor $= S$
    - In this example, $S = 4$
    - For the line, S is always the same as N.
- For the line, $N = S^1$.

Relating Self-Similarity to Dimension:

Dimension 2:

- Begin with a (filled-in) square.
- Divide square into equal sized squares.

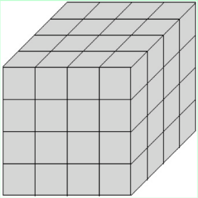


  - Again, each square is identical to the original, just smaller.
  - Number of pieces $=N$ (In this example, $N=16$)
- **Scaling factor** = factor to multiply length of edges by, to make piece identical to original
  - Scaling factor $=S$ (In this example, $S=4$)
- For the square, $N=S^2$.

Relating Self-Similarity to Dimension:

Dimension 3:

- Begin with a (solid) cube.
- Divide cube into equal sized cubes.



  - Again, each cube is identical to the original, just smaller.
  - Number of pieces=N (In this example, $N = 64$)
- **Scaling factor**= factor to multiply length of edges by, to make piece identical to original
  - Scaling factor=S (In this example, $S = 4$)
- For the cube, $N = S^3$.

Relating Self-Similarity to Dimension:

In general:

Begin with a standard geometric figure.

- Let
  - $N=$ number of equal pieces identical to the original but smaller that we cut the original into
  - $S=$scale factor we must multiply each edge by to regain the original,
  - $D=$ dimension of the figure
- Then

$$\boxed{N = S^D}$$

Goal: To find the dimension of a fractal, like the Sierpinski Triangle or the Koch Snowflake

The **defining** property of these fractals has been their self-similarity.

Plan: To relate dimension and self-similarity

With

- N= # of equal pieces identical to original that we cut original into
- S=scale factor we must multiply each edge by to regain the original,
- D= dimension of the figure

Then

$$\boxed{N = S^D}$$

We use this relationship between self-similarity and dimension to expand our idea of dimension. We call this expanded idea of dimension similarity dimension or Hausdorff dimension.