- Exam 2 will *probably* take place on Tuesday 11/6/12, from 5-8pm. (I will not know until campus re-opens and Events and Conferences can process my room request). I will let you know once I know when and where the exam will be.
  - As with Exam 1, I am not *designing* the exam to take 3 hours, but if you used 3 hours last time, it's reasonable to assume you should allow that long for this exam as well.
- If you can not take the exam Tuesday 11/6 from 5-8pm, contact me as soon as possible.
- PS 10 is included at the beginning of this study guide, and – as with Exam 1 – will not be due.
- The exam will cover *from* PS 6 *through* PS 10. Specifically, Exam 2 will cover *from* the Golden Ratio in art, and Fibonacci numbers *through* subdividing and duplicating rectangles.
- The solutions to Problem Sets 6-8 should already be on 2-hour reserve at the circulation desk in the library (along with Problem Sets 1-5). The solutions to the original version of this study guide, with PS 9 included, should also be on reserve. Once campus has re-opened, I will put the solutions to the revised PS 9, as well as the revised Study Guide and PS 10, on reserve.
- I will again be giving you a formula sheet along with your exam. This sheet **will** include:
  - the quadratic formula (just in case)
  - the Golden Ratio
  - the first several Fibonacci numbers
  - Binet's formula

  Not included:
  - the formula for finding a Fibonacci number from the previous two
  - the distance formula for the distance between two points in 2-space *and* in 3-space
  - the formulas for calculating the coordinates of the perspective image of a point – the Perspective Theorem.
  - the Vanishing Point theorem
- Advice:

- Spread studying over several days. Information sinks in better; if you get frustrated, you can take breaks; if some calamity occurs on the day before the exam, you've already done a fair amount of studying; you can get plenty of sleep the night before the exam; etc
- In an ideal world, the best way to study for a math test is to re-read all the readings (including your notes), summarize the topics we've covered, and re-do as many homework problems as possible.

  If don't have time to do all of that,

  * Your main focus should be to *do* as great a variety of problems as possible.
  * In addiiton to doing the problems on this problem set, redo as many problems from problems sets as possible.
  * Reading solutions is not enough.
  * Skim the readings and the notes, emphasizing connections with math and art.
- When you're doing problems, focus on *why* the steps are what they are, and why they make sense. Think about how different problems are connected.
- How long should you study for this? A minimum of 6 hours. If you've struggled with the problem sets, then allow considerably more time.
- If you can not do the problems from start to finish without getting help from friend, tutor, solutions or me, you are not ready. Please note that this does not mean you should *memorize* how to do the problems – the exam will involve similar but not identical ideas. If you *understand* how to do all of these problems as well as all your past homework problems, and can use that understanding to *do* all the problems with no help, then you should be prepared.

• Topics:

- Golden Rectangles & the Extreme and Mean Ratio in art
- What a Golden Triangle is, what it has to do with φ and what it has to do with gnomons
- Fibonacci numbers
- How the Fibonacci numbers are related to φ
  * sequence of $\frac{F_n}{F_{n-1}}$
  * Binet's formula
  * anything else you can think of

- Using Binet's formula
- The distance formulae for points in 2-space and for points in 3-space
- Plotting points in 3-space, and how to leave clues as to where that point lies (the box).
- The relationship between points in 3-space (for instance, as in our cube problems)
- The Perspective Theorem-where it comes from, and how to use it
- The meaning of the word "orthogonal"
- Vanishing points
  - what is a vanishing point?
  - where do images of lines orthogonal to the picture plane vanish?
  - How about lines parallel to the picture plane (the xy-plane)?
  - Lines parallel to the "floor" (the xz-plane)?
  - Lines parallel to a "side wall" (the yz-plane)?
- Vanishing points of parallel lines – finding them on a picture.
- Finding the correct viewing position for a drawing in one-point perspective.
- The rules of perspective
- Subdividing rectangles in perspective, in half, quarters, eighths – any power of 2.
- Duplicating rectangles in perspective, so that the original and the copy share an edge.
- Subdividing rectangles in perspective, in thirds, fifths, sixths – any number of subdivisions that is **not** a power of 2.
- Duplicating rectangles in perpsective, so that the two do not share a common edge, if you know where you want to put the *near* edge of the copy.
- Duplicating rectangles in perspective, so that the two do not share a common edge, if you know where you want to put the *far* edge of the copy.

Problem Set 10 Problems:

For all of these problems, print outcopies of the drawing of a section of a roadside fence (the link is with the link for this study guide)

1. Within the solid outline of the fence section, draw 7 equally spaced vertical fenceposts to create a fence with 8 equal sections.
2. Extend the fence into the distance by drawing three exact perspective duplicates of the original rectangular section, each attached to the far side of the previous.

3. Treating the given solid outline as one section of fence, draw a copy that is a duplicate of the original –with the top of its *nearest* fencepost occurring at the point P. (In other words, there should be a space between the two sections of fence).

   Explain (mathematically) why this technique works.

   Note: It is just a coincidence that P is close to where a diagonal through the midpoint of the side hits; P could be *anywhere* along the top rail. The idea here is to find a technique that allows us to draw an exact copy of a rectangle with any-sized space in between the two, by choosing where to put the nearer edge of the rectangle.

4. Draw a duplicate of the section of fence (in the same plane as the original), this time with the top of its *far* fencepost at the point P.

   Explain (mathematically) why this technique works.

   Note: Again, assume that there is nothing special about where P is, it could be anywhere on the extension of the top fence rail. Your new rectangle may or may not overlap with the original section of fence. Being able to draw two overlapping identical rectangles in perspective comes in handy when drawing such things as a partially open sliding glass door or window, for instance.

5. Draw 2 vertical fenceposts to divide the fence into 3 equal sections. (This may seem easy at first, but 3 equal sections is much different from 2, 4, 8, 16 etc.)

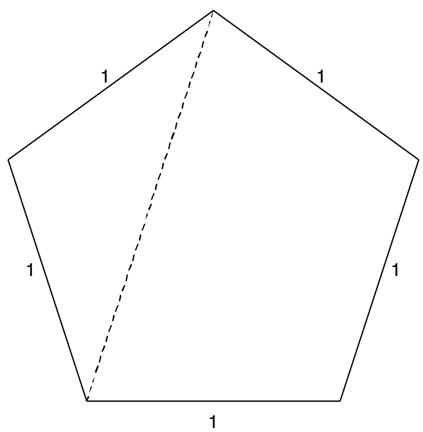
   Explain (mathematically) why this technique works.

Study Guide Problems:

The following problems are intended as a supplement to your review; they are not intended to replace reviewing the reading and class notes, or redoing homework problems.

Remember: You are responsible for all material covered in your reading, whether or not we covered it in class.

1. The regular pentagon in the following figure has sides of length 1. Use the fact that the angle a diagonal forms with the closest side of the pentagon is 36°, along with the results of a problem from PS 6, to show that the length of any one of its diagonals is φ.



2. Use that $F_{26} = 121\ 393$ and that $F_{28} = 317\ 811$, to find F_{29}.

3. Let a represent the 300th Fibonacci number and b represent the 301st Fibonacci number. Express the 298th Fibonacci number in terms of a and b. Simplify your answer.

4. Claim: $(F_1 + F_2 + F_3 + \ldots + F_N) + 1 = F_{N+2}$. Verify this claim for:

   (a) $N = 4$

   (b) $N = 10$

   Hint: Verify means show that the claim really *is* true when $N = 4$, or $N = 10$. For instance, to verify that the claim is true for $N = 4$, you need to verify that the left side is equal to the right side. One approach would be to figure out what the left side *is*, and do the same thing for the right side. If they're equal, you have verified the statement.

5. *Calculating powers of φ.*

   Remember that φ is one of two solutions to $x^2 - x - 1 = 0$ $\left(\frac{1-\sqrt{5}}{2} \text{ is the other}\right)$. Of course, this means that $\varphi^2 - \varphi - 1 = 0$, or in other words, that $\varphi^2 = \varphi + 1$.

   (a) Verify that $\varphi^2 = \varphi + 1$ by comparing φ^2 and $\varphi + 1$ on your calculator!

   (b) Use that $\varphi^3 = \varphi^2 \cdot \varphi$, along with the above relationship, to show that $\varphi^3 = 2\varphi + 1$. *Hint:* Replace φ^2 with $\varphi + 1$, simplify, and replace φ^2 with $\varphi + 1$ again.

(c) Use your result for φ^3, along with the fact that $\varphi^4 = \varphi^3 \cdot \varphi$, to show that $\varphi^4 = 3\varphi + 2$ – follow a similar strategy as you did in the previous part.

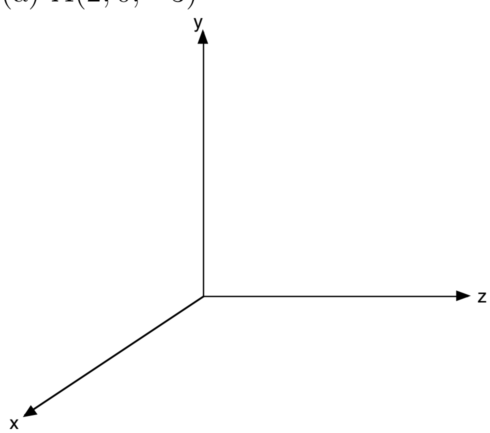
(d) Show that $\varphi^5 = 5\varphi + 3$.

(e) Look for a pattern in the results for φ^2, φ^3, φ^4, and φ^5. Based on what you see, what do you think φ^6 is ? Check your results.

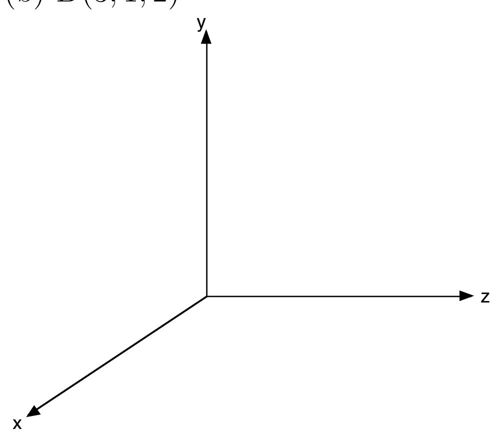
(f) In general, how do you think φ^N can be rewritten, in terms of just a single power of φ and some whole numbers?

6. Plot the following points on a set of 3-D coordinate axes. Leave in lines indicating where in space your points are located.

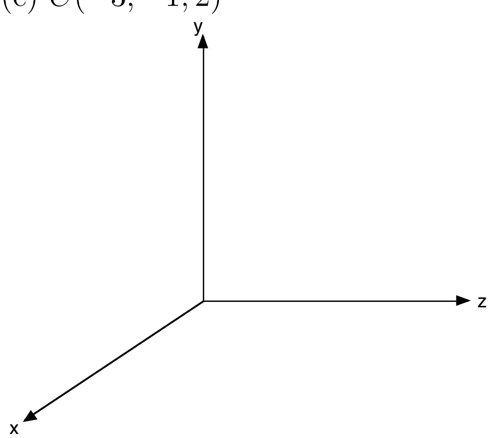
(a) $A(2, 0, -3)$



(b) $B(3, 1, 2)$



(c) $C(-3, -1, 2)$



7. A cube is placed so that its faces are parallel to the coordinate planes. The length of each edge is 7, and to a viewer located on the negative z-axis, the bottom left front corner has coordinates $(1, -3, 2)$.

   (a) What are the coordinates of the other seven corners of the cube?

   (b) Use the Perspective Theorem to find the perspective image of each of the eight corners. Use a viewing distance of 2 units.

   (c) Carefully draw the cube in perspective, using the images of each coordinate that you found in the previous part.

8. From OnCourse, with this study guide, print out a copy of Piero della Francesca's *The Flagellation*. On it,

   (a) Locate the primary vanishing point.

   (b) If there are any secondary vanishing points, find one.

   (c) Determine the ideal viewing position.

9. From OnCourse, print out a copy of Fra Francesco Colonna's *Garden of Love* from his *Hypnerotomachia Poliphili*. (It's small, do the best you can with it.)

   (a) Locate the primary vanishing point (as closely as you can).

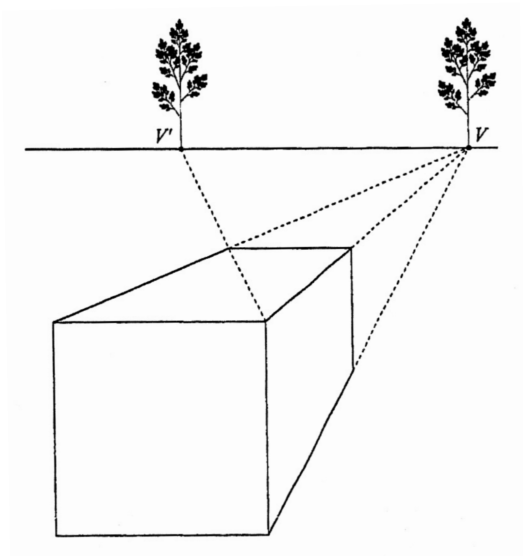
   (b) The two benches on the central platform each have vertical rectangular sides that are parallel to side walls. Assuming they are the same size, their diagonals should be parallel in real life, and hence the images of the diagonals should have the same vanishing point. Where should that vanishing point lie? Check it out – does it (or is it at least close?)

   (c) Similarly, the lines forming the diamonds on the side fences should form two sets of parallel lines parallel to side walls. What *should* be true about their vanishing points? Again, investigate whether it *is* true.

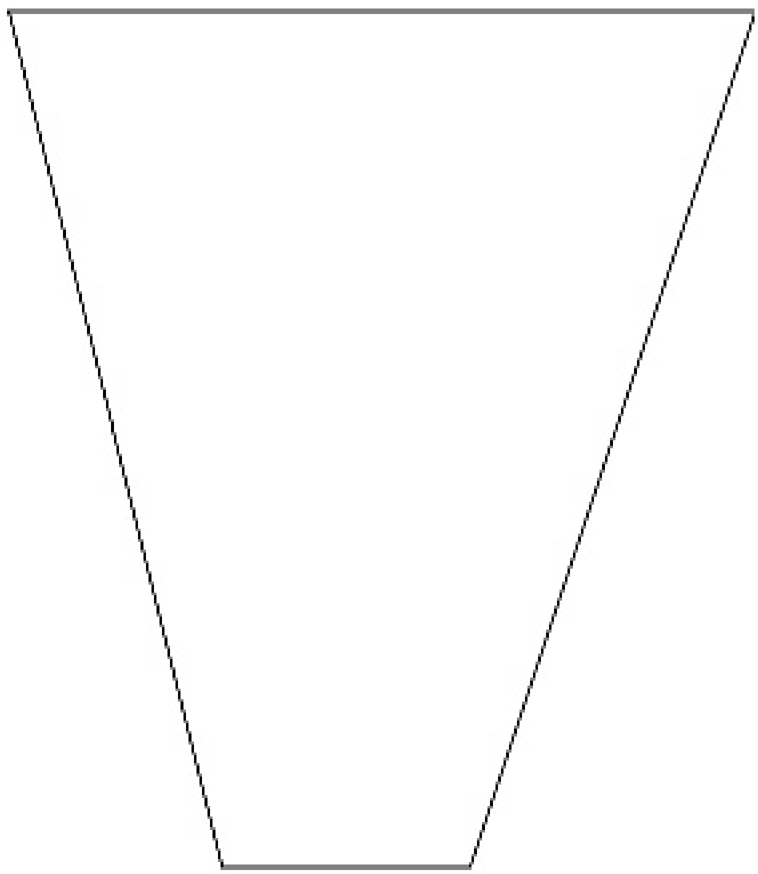
10. Draw a section of tile wall so that

    - the wall is a side wall extending orthogonally away from the picture plane, *and*
    - the tiles are all square, and the same size, *and*
    - the section of wall that you draw is 5 tiles deep and 5 tiles high, *and*
    - tiles are oriented with front & back edges parallel to the picture plane, while top & bottom edges are orthogonal, *and*
    - The viewing distance is 8"

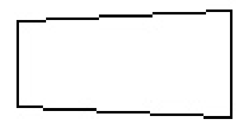
11. If the box below represents a cube, then we can use our usual techniques to find the correct viewing position. But suppose the box is *not* a cube. Suppose instead that for whatever reason we know that the *side* of the box is intended to be three times as deep (that is, from front to back) as it is *tall*. Using similar ideas to those from class and the reading, determine what the viewing distance in this case is (from scratch).



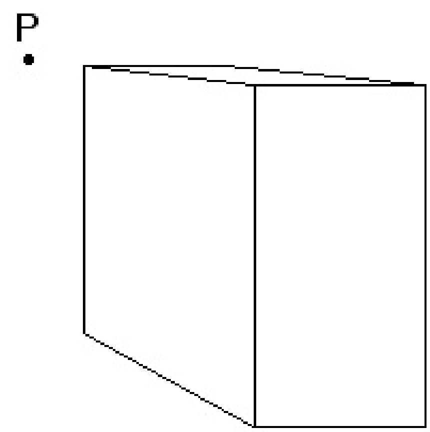
12. Divide the perspective drawing of a rectangle below in half lengthwise, without measuring. (That is, draw a line that cuts the lines which no longer appear parallel in half.) Then divide the nearer of your halves in half; the nearer of your quarters in half, and the nearer of your eighths in half. In the end, the rectangle should have one half, one fourth, one eighth and two sixteenths.



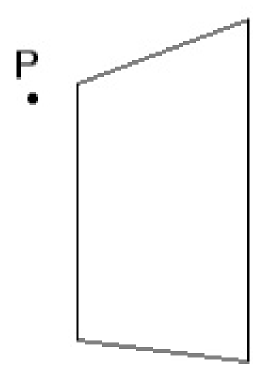
13. Beginning with the rectangle shown below, which represents one brick, draw a portion of a brick wall consisting of identically sized bricks (in real-life) that is 3 bricks wide and 4 bricks high. Remember that in order to do this so it really looks like a brick wall, the second row of bricks must be offset from the first row, so that the end of one brick divides the brick below it in half.



14. Below is a perspective drawing of a box, along with a point P. Draw a duplicate of the box, using the techniques we've developed. Place the duplicate so that it's front left corner (as we face it) is located at the point P, to create a picture of two boxes separated by some space.



15. Below is a perspective drawing of a window, retreating orthogonally to the picture plane. Draw a duplicate of this window, so that its upper rear corner is located at the point P, to create the appearance of a partially open sliding glass door.



16. On the perspective drawing of a rectangle below, draw a horizontal line cutting the sides which no longer appear parallel into the division one-ninth/eight-ninths, without measuring. Probably the easiest way to do this is to divide the rectangle into thirds, and then one of the thirds on an end into thirds again.