- REMINDER: The final is from 2-5, Wednesday 12/12, in our usual classroom, Mars SC 1141. Plan on it taking the entire 3 hours.
- *Note:* I may allow people who want to to begin at 1pm.
- The final *will* be cumulative.
- The solutions to Problem Set 13 will soon be on 2-hour reserve at the circulation desk (along with the already-available solutions to Problem Sets 1-4, 6-9, and 11-12; the solutions to Problem Sets 5 and 10 are a part of the solutions to Study Guide 1 and Study Guide 2, respectively; those solutions are also still available). Solutions to this study guide should be available by Monday of finals week.
- CHEAT SHEET: You may use a 4×6 (or smaller) index card for notes. You can put anything you want to on it. I will still include the Vitruvian proportions and the Sacred Cut ratios on the exam, because they take up a lot of space, but everything else is up to you.
- WARNING: As usual, this is not intended to be a stand-alone study resource. In fact, this only covers the most recent material; use old study guides and past exams as a starter for the older material.
- ADVICE:
  - Spread studying out over the next several days
  - Look over what topics we covered, then review your notes and the reading. But as always, the single most effective way of studying is doing as many problems as possible. The problem sets and studyguides for Exams 1 through 3 are still on the course web page, if you'd like fresh copies.
  - Remember to think about *why* each step is true. Ask yourself if each step makes sense.
  - How long should you study for this? I'd suggest an absolute minimum of 9 hours. If you've struggled with past exams, allow more time.
  - Remember: If you can not do the problems from start to finish without getting help from friend, tutor, solutions or me, you are not ready. Please note that this does not mean you should *memorize* how to do the problems – as you know from the midterm exams, the final will involve similar but not identical ideas. If you *understand* how to do all of these problems as well as all your past homework problems, and can use that understanding to *do* all the problems with no help, then you should be prepared.
- TOPICS:

- All the topics mentioned in the previous three study guides, still available on-line,

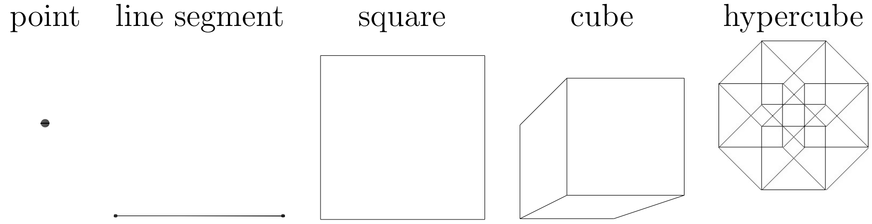
  PLUS:
- What a Linelander would see when a 2 dimensional object passes through its space, what a Flatlander would see when a 3 dimensional object passes through its space, and what we would see when a 4 dimensional object passes through our space. (I would only ask this for the appropriately dimensioned analog of the cube or sphere). By *see*, I really mean *experience through a combination of touch and sight, if given enough time to move around and feel as much as the dimensional restrictions allow.*
- Looking for patterns in the number of vertices, edges, faces, solids, etc in a point, line segment, square or triangle, cube or tetrahedron, and using the patterns to predict how many vertices, edges, faces, solids, 4-d regions and 5-d regions are in the hypercube, hyperhypercube, 6th dimensional cube, etc or hypertetrahedron, hyperhypertetrahedron, 6th dimensional tetrahedron, etc.
- Ways to visually represent the hypercube in 2 and 3 dimensions.
- The connection between the 4th dimension, non-Euclidean geometry, and early 20th century art – who were some of the artists that we know were influenced by these mathematical notions? (Juan Gris, Jean Metzinger, Albert Gleizes, Marcel DuChamp, and Max Weber are the ones whose work we saw in class. **Not** Picasso.)

• PROBLEMS:

The following problems are intended as a supplement to your review; they are not intended to replace reviewing the reading and class notes, or redoing homework problems.

1. What would A. Square observe (assuming he has the leisure to walk around and perhaps even touch), if a cube passed through Flatland edge first?
2. Recall the creation of a line segment, square, cube, and hypercube:

0D	1D	2D	3D	4D
point	line segment	square	cube	hypercube



Pondering the creation of the hypercube and the hyperhypercube resulted in the following table, up through dimension 5. Complete the final column for the sixth dimensional hyper-hyper-hypercube. You may use formulas we developed or the process of creating the hyperhyperhypercube, but not *just* patterns in the table.

However you do it, give some brief explanation that involves the creation of the hyperhypercube.

Dimension Figure	0D point	1D line segment	2D square	3D cube	4D hypercube	5D hyper-hypercube	6D hyperhyper-hypercube
vertices v	1	2	4	8	16	32	
edges e	0	1	4	12	32	80	
faces f	0	0	1	6	24	80	
solids s	0	0	0	1	8	40	
4D regions t	0	0	0	0	1	10	
5D regions u	0	0	0	0	0	1	
6D regions w	0	0	0	0	0	0	

3. We can define something called the *Euler characteristic* of any figure in any dimension. For figures in six dimensions or less, the Euler characteristic is defined as follows:

$$\text{Euler characteristic} = \chi = v - e + f - s + t - u + w,$$

where v represents the number of vertices of the figure, e the number of edges, f the number of faces, s the number of solids, t the number of 4-dimensional regions, u the number of 5-dimensional regions, and w the number of 6-dimensional regions.

Be aware: The Euler characteristic for the outline of a square (which has vertices and edges but not faces) is different from that of a filled-in square; similarly, the Euler characteristic for a shell of a cube is different from that of a cubical solid. The table that you have created is assuming a filled-in square, a cubical solid, a filled in (whatever that means) hypercube, etc.

Find the Euler characteristic χ of each of the figures listed in the table in the previous problem. Notice anything?

4. Use ideas and/or formulas developed in class to decide what the most likely intersection of a plane with a 3-space is in 4-space.