1. (Exercise 9 in Section 6.2) We have seen in class and in homework how we can determine the number of vertices v, edges e, faces f, solids s, 4-dimensional regions t and 5-dimensional regions u that are in a hypercube and a hyperhypercube. The results we've found are summarized in the table below:

Dimension Figure	0D point	1D line segment	2D square	3D cube	4D hypercube	5D hyperhypercube
vertices v	1	2	4	8	16	32
edges e	0	1	4	12	32	80
faces f	0	0	1	6	24	80
solids s	0	0	0	1	8	40
4D regions t	0	0	0	0	1	10
5D regions u	0	0	0	0	0	1

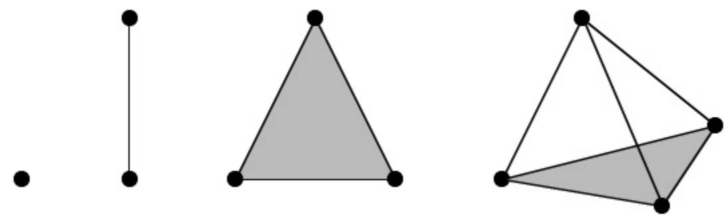
We can define something called the *Euler characteristic* of any figure in any dimension. For figures in five dimensions or less, the Euler characteristic is defined as follows:

$$\text{Euler characteristic} = \chi = v - e + f - s + t - u.$$

Find the Euler characteristic χ of each of the figures listed in the above table. Notice anything?

2. (An adaptation of exercise 14 in Section 6.2) Cubes, hypercubes, hyperhypercubes etc are all higher-dimensional analogies of the two-dimensional square. On page 203-204, your text discusses how other higher-dimensional figures can be built up in analogy with the triangle:

> *Start with a single point. In the next generation, add another point above the first, and connect the two, creating a line segment. Place that line segment on the floor and add another point above the line segment and connect this point with each of the points on the line segment, obtaining a triangle. Place the triangle flat on the floor and another point above it; connecting the new point to each of the points in the triangle gives a three-dimensional solid called the tetrahedron or triangular pyramid.*



Adding another point in the fourth dimension, ana *or* kata *the figure, and connecting this point to each point in the tetrahedron gives a figure called the* hypertetrahedron *or* pentahedroid.

Develop analogous formulae to those we developed for the hypercube and the hyperhypercube to fill in the table below.

Dimension Figure	0D point	1D segment	2D triangle	3D tetrahedron	4D hypertetrahedron	5D hyperhypertetrahedron
vertices v						
edges e						
faces f						
solids s						
4D regions t						
5D regions u						

3. Find the Euler characteristic $\chi = v - e + f - s + t - u$(first defined above) for each of the n-dimensional triangular figures in the previous exercise.

4. (Exercise 7 in Section 11.1) What can you say about the area of a circle in elliptic space? In hyperbolic space?