1. For each of the following, you'll be drawing a line that is cut in mean and extreme ratio (i.e. the Golden Ratio).

   (a) Suppose we want to draw a line cut in mean and extreme ratio, and we want the short segment to be 3 units long. How long should the long segment be? Draw such a line as carefully as possible.

   (b) Suppose we want to draw a line of length 6 that is cut in mean and extreme ratio. Where should we place the cut? Draw such a line, as carefully as possible.

   Look at the two lines with cuts that you've drawn. Do they look the same or different? (You don't necessarily have to address this in what you turn in, I just want you to pay attention to the big picture!)

2. Suppose we don't happen to agree with Euclid and all those crazy Greeks about balance between extremes. So we define our own standard of beauty and all that, beginning with the ideal way to cut a line. Here's ours:

   > *A line is said to be cut in a* **very cool** *ratio when the greater segment is to the lesser segment as twice the whole is to the greater.*

   (a) What *is* this very cool ratio? (By this I mean, *find* the number it equals!)

   (b) Suppose we want to draw a line cut in a very cool ratio, and we want the short segment to be 3 units long. How long should the long segment be? Draw such a line as carefully as possible.

   (c) Suppose we want to draw a line of length 6 that is cut in a very cool ratio. Where should we place the cut? Draw such a line as carefully as possible.

   (d) How do these cuts compare to those you found in the previous problem?

3. You want to paint a picture, and you want the canvas to be in the shape of a Golden Rectangle – that is, you want the ratio of the length of the long side to the length of the short side to be φ. If the short side of your canvas is 2' wide, how long should you make it be?

4. You have read, in the excerpts from *The DaVinci Code*, that "my friends, each of you is a walking tribute to the Divine Proportion." In this exercise, you will explore whether *you* are a such a tribute to the Golden Ratio.

   (a) Measure the following. In each case, give an accuracy range.

      i. your height
      ii. the height of your belly button
      iii. the distance from your shoulder to your fingertips
      iv. the distance from your elbow to your fingertips
      v. the distance from your hip to the floor
      vi. the distance from your knee to the floor

      For instance, you might list your height as $5'3.5" \pm .5" = 63.5" \pm .5"$. Just as I did in this example, make sure your lengths are not in mixed units like feet and inches, but just in inches (or centimeters). Use the same units throughout your measurements.

   (b) *Height to belly button height:*

      i. Calculate the ratio of your height to the height of your belly button.
      ii. Calculate the range that this ratio might fall into, using the accuracy ranges you found above.
      iii. Does the Golden Ratio fall into this range?

   (c) *Arm length to fore-arm length:*

      i. Calculate the ratio of your arm length to your fore-arm length.
      ii. Calculate the range that this ratio might fall into.
      iii. Does the Golden Ratio fall into this range?

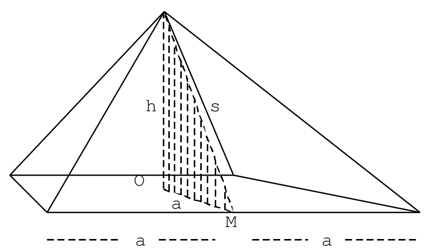
   (d) *Leg length to height of knee:*

      i. Calculate the ratio of your leg length to the height of your knee.
      ii. Calculate the range that this ratio might fall into.
      iii. Does the Golden Ratio fall into this range?

   (e) Draw some conclusions as to whether you believe *you* are the tribute to the Divine Proportion that Dan Brown's Robert Langdon claims you are.

5. You have read, in *Under the Starry Pointed Pyramid*, that it is frequently said that Herodotus described the construction of the Great Pyramid by saying that the Pyramid was built so that the area of each face would equal the area of a square whose side is equal to the Pyramid's height. You have also read that if Herodotus had indeed said this, it would have meant that the Golden Ratio was sure to appear in the Great Pyramid, but Livio left out some details.

   In this exercise, we will explore this idea in more detail.



   (a) Using h for the height of the pyramid, as shown in the above figure, what would be the area of a square whose side is equal in length to the Pyramid's height?

   (b) What is the area of one of the triangular faces of the Pyramid? Use a for the length of half the base, and s for the "slant-height" (the height of a face, as measured on the face), as is used in the above figure.

   (c) Rewrite the statement attributed to Herodotus, using the expressions for area you found in the previous two exercises.

   (d) By looking at the above diagram of the pyramid, find another equation that connects h, s, and a.

   (e) Combine these two equations in a logical way to find a relationship between a and s. Solve for s/a. (You should get that $s/a = \varphi$!)

6. Recall from class February 7th, and from your reading in Chapter 9, that the equation $x^2 - x - 1 = 0$ has two solutions:

$$\begin{aligned} s_1 &= \frac{1+\sqrt{5}}{2} \approx 1.618033988 \\ s_2 &= \frac{1-\sqrt{5}}{2} \approx -.6180339880 \end{aligned}$$

(The first one gives the Golden Ratio).

Explain why these two solutions have the same decimal expansions.

Hint: One property of quadratic equations is that if s_1 and s_2 are the two solutions to $ax^2 + bx + c = 0$, then $s_1 + s_2 = -b$.

7. In the following figure, $ABCD$ is a square, and the three triangles I, II, and III have equal areas. Using some geometry and some algebra, show that x/y is the Golden Ratio.