These problems are all either from Symmetry, Shape and Space, *chapter 6: "Other Dimensions, Other Worlds",* adaptations of problems from that chapter, or inspired by that chapter.

1. (Exercise 7 from Chapter 6, Section 1: Other Dimensions, other worlds; Flatlands) A. Square has a conversation with his grandson.

   > 'I began to show the boy how a Point by moving through a length of three inches makes a Line of three inches, which may be represented by 3; and how a Line of three inches, moving parallel to itself through a length of three inches, makes a Square of three inches every way, which may be represented by 3^2 [square inches].'

   Generalize to give a geometric significance to the quantity 3^3, thus answering a question posed by the grandson.

2. What would A. Square observe (assuming he has the leisure to walk around and perhaps even touch), if a cube passed through Flatland:

   (a) with its bottom parallel to the plane of Flatland?

   (b) with one corner approaching Flatland before the rest? (This one is tricky – do the best with it you can!)

3. What would a 4-dimensional cube look like to us if it passed through our 3-dimensional space "flush" with our space? (I'm of course having trouble finding words to describe, since we don't have many words to describe hyperspace, but what I'm asking you to do is the analogy of part (a) in the previous problem). Describe and draw pictures as necessary to make your point.

4. (An expansion of Exercise 14 in Section 6.1) Think about moving the hypercube of four dimensions parallel to itself to form a 5-dimensional *hyperhypercube.*

   (a) How many vertices does the hyperhypercube have, and why? (When answering the why part of the question, discuss this in the context of how the hyperhypercube is formed.)

   (b) How many edges does the hyperhypercube have, and why? (Again, discuss in the context of how its formed.)

(c) How many 2-dimensional faces does the hyperhypercube have, and why? (Same type of discussion as above.)

(d) How many 3-dimensional solids does it have? Justify.

(e) How many 4-dimensional regions? Justify.

(f) How many 5-dimensional regions? Justify.

5. (Exercise 3 in Section 6.2) In the plane, note that any two non-parallel lines intersect in a point.

   (a) Generalize this idea to the intersection of two non-parallel planes in the third dimension. That is, what does the intersection of any two non-parallel planes look like?

   (b) Generalize even further, this time to the intersection of two non-parallel three-dimensional spaces in hyperspace.

6. (Exercise 4 in Section 6.2) In the illustration on the left below, you see that in general, (that is, usually) a point and a line on the plane will not intersect at all. In other words, given a randomly chosen line and a randomly chosen point in the same plane, more likely than not the point will not be on the line. In the illustration shown on the right, you see that usually a line and a plane in 3 dimensional space will intersect in a point.
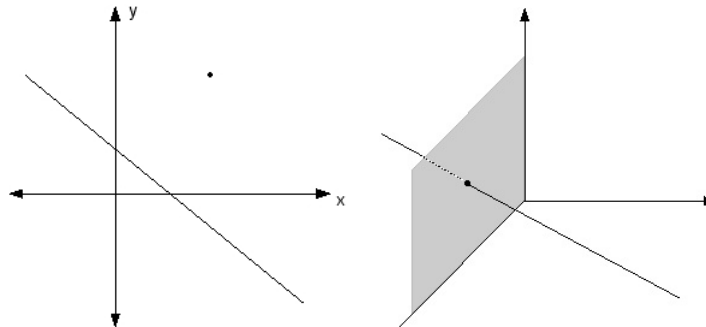
(Of course, this does not always occur: the line could be parallel to the plane, or the line could lie on the plane, but given an randomly chosen line in space and a randomly chosen plane in space, *more often than not* the line will intersect the plane in only one point.)

(a) What is the most likely intersection of two lines in the plane? *Draw a sketch to illustrate.*

(b) What is the most likely intersection of two lines in 3-space? *Draw a sketch to illustrate.*

(c) *Looking for patterns:* Look at your results so far for this exercise, as well as those for part (a) of the previous exercise. In each case, you were looking at the intersection of either a line (dimension 1) or plane (dimension 2) with a line (dimension 1) or plane (dimension 2), and in each case, the intersection occurred in either the plane (dimension 2) or 3-space. And in each case, the resulting intersection was either nothing (no dimension), a point (dimension 0), or a line (dimension 1).

   i. Make a table comparing the dimensions of the two intersecting objects, the dimension of the space in which the intersection occurs, and the dimension of the most likely resulting intersection.

   ii. Find a relationship (a formula) that relates the dimension of the resulting intersection with the dimensions of the two intersecting objects and the dimension of the space.

(d) What is the most likely intersection of a line and a plane in hyperspace? Justify your answer using whatever balance of insight into the 4th dimension and use of the previous result works for you.

(e) What is the most likely intersection of two planes in hyperspace? Again, justify your answer

(f) What is the most likely intersection of a plane and a 3-space in hyperspace?

7. (Exercise 6 in Section 6.2) In class, we have tossed about the idea that if we could move in the 4th dimension, we could remove an object from a sealed box. (It was this sort of idea that connected thoughts of the 4th dimension with magic). More formally explain why this would be true, using some of our extrapolations of behavior in the 4th dimension.

Hint: Think of the wall of a box as a plane. The movement of a hand (with or without an object in it) is linear. Also, think of exercises you've already done for this problem set!