- WARNING: Please don't take this as the final word on how to study. First of all, everybody learns differently, second of all, I am an expert at math, not at the theory of studying, and finally, I'm squeezing this in among all the *other* things I have to get done (much as you're doing with all of your work), and so I may not think of everything. I would hate for someone to follow my advice to the letter (perhaps against their better judgment) and have it not work for them.

- ADVICE:

  - Let me emphasize this again –I know you have other classes, but spread studying for this exam out over several days. Information sinks in better; if you get frustrated, you can take breaks; if some calamity occurs on the day before the exam, you've already done a fair amount of studying; you can get plenty of sleep the night before the exam; etc

  - In an ideal world, the best way to study for a math test is to re-read all the readings (including your notes – this course is definitely heavily notes-based!), summarize the topics we've covered, and re-do as many homework problems as possible.

    If you are not living in an ideal world (and who is), I would still skim the readings, and in the notes from class try to emphasize connections with math and art that may not have been covered much in the readings. Your main focus, however, should be to *do* (not to read) as great a variety of problems as possible. In addition to doing the few problems I've included on this study guide, you'll also want to redo as many problems as you can from the first three problem sets. (Notice that I said "redo" – simply reading through solutions doesn't do it.)

  - When you're doing problems, focus on *why* the steps are what they are. Spare some of your thoughts for how different problems are connected, and why various steps make sense.

  - When doing a problem that you've done before, don't waste your time trying to remember how you did before–often, memory proves to be false and can lead you astray. Just focus on doing what makes sense.

  - Should you study alone or with other people? That varies from person to person, but in general I'd say most of your studying should be on your own, particularly as it gets closer to the day of the exam. I think group study is best for most people at the beginning of the study process. Since the exam is individual, at some point in your studying, you have to be doing problems individually.

  - How long should you study for this? Alot. "Alot" will vary from person to person also, but I'd suggest an absolute minimum of 6 hours. If you've struggled with the problem sets, then leave more. If you breezed through the problem sets, then you *may* be able to get away with less – but why risk it?!

- Topics:
  - Similar figures
  - Gnomons
  - How the Golden Ratio can be derived from the notion of gnomons
  - What a Golden Rectangle is, what it has to do with φ and what it has to do with gnomons
  - What a Golden Triangle is, what it has to do with φ and what it has to do with gnomons
  - Fibonacci numbers
  - How are the Fibonacci numbers related to φ
    - sequence of $\frac{F_n}{F_{n-1}}$
    - Binet's formula
    - Powers of φ
    - anything else you can think of
  - Using Binet's formula
  - How/where the Golden Ratio shows up in a pentagon/pentagram.
  - The distance formulae for points in 2-space and for points in 3-space
  - Plotting points in 3-space
  - The relationship between points in 3-space (as in all our cube problems)
  - The Perspective Theorem-where it comes from, and using it
  - The meaning of the word "orthogonal"
  - Vanishing points - where do images of lines orthogonal to the picture plane vanish? How about lines parallel to the picture plane (the xy-plane)? Lines parallel to the "floor" (the xz-plane)? Lines parallel to a "side wall" (the yz-plane)?
  - Vanishing points of parallel lines
  - Finding the correct viewing position for a drawing in one-point perspective.

- Problems:

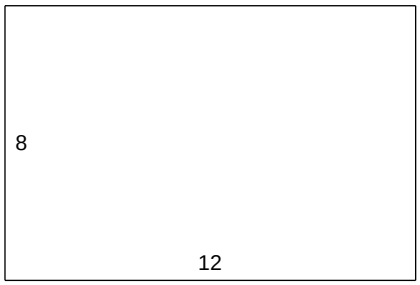
  The following problems are intended as a supplement to your review; they are not intended to replace reviewing the reading and class notes, or redoing homework problems.

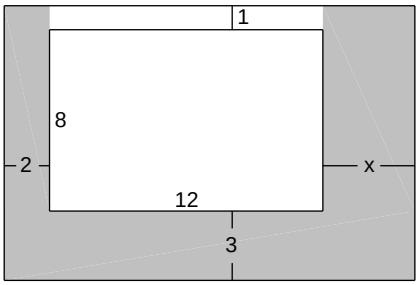
  A word of caution: You are responsible for all material covered in your reading, whether or not we covered it in class.

1. We have shown in class that a pyramid created by measuring the base using some number of revolutions of the drum, and the height by using twice that number of diameters of the drum is very close to being similar to the Great Pyramid at Gizeh.

   While the Great Pyramid is the most famous of the pyramids, there are others at Gizeh (as well as throughout Egypt). The dimensions for the Second Pyramid at Gizeh are 470.75 feet high, with the sides of the base each being approximately 702' long. Is this pyramid also close to being similar to a pyramid that is constructed using the above technique?

2. *Number 29 from* Excursions in Modern Math*, Chapter 9*: Find the value of x so that the shaded "rectangular ring" is a gnomon to the white rectangle.





3. *Number 26 from* Excursions in Modern Math*, Chapter 9*: Rectangle A is 2 by 3. Rectangle B is a gnomon to rectangle A. What are the dimensions of rectangle B?

4. *Number 44 from* Excursions in Modern Math*, Chapter 9*: A rectangle has a square gnomon. The new rectangle obtained by attaching the square gnomon to the original rectangle has longer leg 20. What are the dimensions of the original rectangle?

5. *Inspired by Number 6 from* Excursions in Modern Math*, Chapter 9*: Use that $F_{26} = 121,393$ and that $F_{28} = 317,811$, find F_{27}.

6. *Also inspired by Number 6 from* Excursions in Modern Math*, Chapter 9*: Use that $F_{21} = 10,946$ and that $F_{23} = 28,657$, find F_{20}.

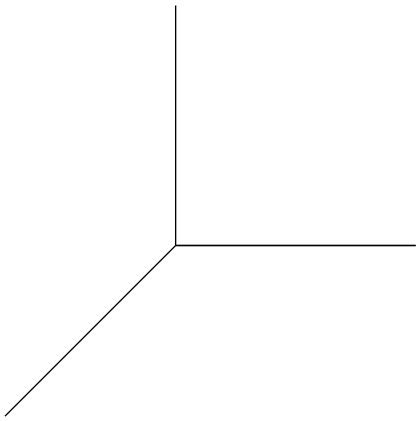
7. *Inspired by Number 13 from* Excursions in Modern Math*, Chapter 9*: Fact: $(F_1 + F_2 + F_3 + \ldots + F_N) + 1 = F_{N+2}$. Verify this fact for:

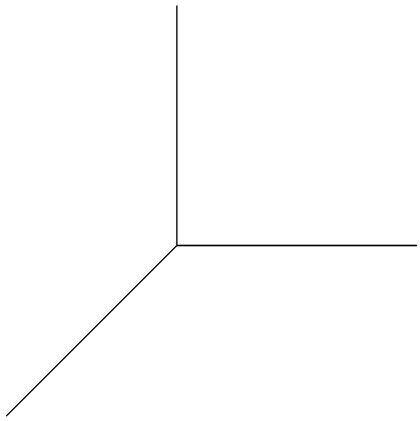
(a) $N = 4$

(b) $N = 10$

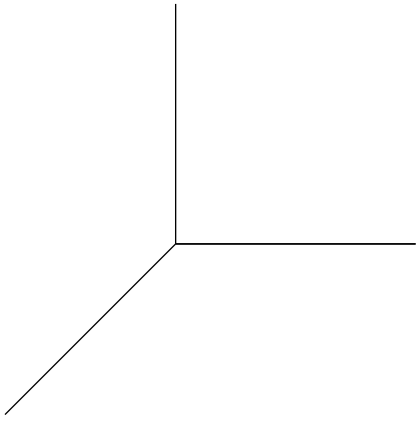
8. Please plot the following points on a set of 3-D coordinate axes.

(a) $A(2, 0, -3)$ (b) $B(3, 1, 2)$





(c) $C(-3, -1, 2)$



9. Consider a box whose faces are parallel to the coordinate planes. Suppose the coordinates of two opposing corners of the box have coordinates $(-7, -10, 5)$ and $(-3, -4, 13)$.

(a) How wide is the box in the x direction?

(b) How tall is the box in the y direction?

(c) How deep is the box in the z direction?

(d) What are the coordinates of the remaining 6 corners?

(e) Use the Perspective Theorem to find the coordinates of the perspective image of each of the eight corners. Use a viewing distance of 10 units.

(f) Plot the images you found in the previous part in the xy plane. Be careful, and use graph paper . Use straight lines to connect the corners that are connected on the real box (of course, use dashed lines for the hidden edges.)

10. Suppose this time we have a cube whose faces are again parallel to the coordinate planes, but only the coordinates of one corner are known. Suppose the bottom left back corner has coordinates $(1, -3, 2)$ and that the length of each edge is 7.

(a) What are the coordinates of the other seven corners of the cube?

(b) Use the Perspective Theorem to find the perspective image of each of the eight corners. Use a viewing distance of 2 units.

(c) Plot the images you found in the part in the xy plane, on graph paper.

11. For this problem, you will need to print out from the web (or photocopy from a book) a picture of Piero della Francesca's *The Flagellation*. Here is one website that has it:
`http://www.wga.hu/frames-e.html?/html/p/piero/francesc/flagella.html`
(If you click on the image, you can see a larger version.)

(a) Locate the primary vanishing point.

(b) If there are any secondary vanishing points, find one.

(c) Determine the correct viewing position.

12. If the box below represents a cube, then we can use our usual techniques to find the correct viewing position. But suppose the box is *not* a cube. Suppose instead that for whatever reason we know that the *side* of the box is intended to be twice as deep from left to right (that is, from front to back) as it is tall. What is the viewing distance in this case?