



- REMINDER: The final is from 2-5, Saturday 5/9, in our Knaption 114. It *will* be cumulative.
- CHEAT SHEET: You may use a  $4 \times 6$  (or smaller) index card for notes. You can put anything you want to on it. I will still include the Vitruvian proportions and the Sacred Cut ratios on the exam, because they take up a lot of space.
- WARNING: As usual, I am squeezed for time as I write this, so please do not assume that it is a stand-alone study resource.
- ADVICE:
  - Spread studying out over the next several days
  - Look over what topics we covered, then review your notes and the reading. But as always, the single most effective way of studying is doing as many problems as possible. Fresh copies of the problem sets or studyguides are on the course web page.
  - Remember to think about *why* each step is true. Ask yourself if each step makes sense.
  - How long should you study for this? I'd suggest an absolute minimum of 9 hours. If you've struggled with past exams, allow more time.
- TOPICS:
  - All the topics mentioned in the previous three study guides, available on-line,  
PLUS:
    - Understand the idea of symmetry of scale
    - Understand the recursive construction of the Koch Snowflake, the Sierpinski Triangle, the Sierpinski Carpet, and the Mitsubishi Gasket.
    - Be able to do the first several steps of creating a new fractal using similar geometric/visual recursion steps to those used to the above-mentioned fractals.
    - Understand how we found the dimension of the Koch Snowflake, the Sierpinski Triangle, and the Sierpinski Carpet.
    - Know how to use the recursion/iteration to find whether a point belongs in the Mandelbrot set or not. Know what a seed is, and what *escaping*, *attracted*, and *periodic* means in relation to the Mandelbrot set. Know what a Mandelbrot sequence is.
- PROBLEMS:

The following problems are intended as a supplement to your review; they are not intended to replace reviewing the reading and class notes, or redoing homework problems.

1. *The Checkered Flag*

- (a) Using graph paper, carefully draw the figure that results after the first three steps, with the following recursive replacement rule:

Start with a white rectangle . Whenever (and wherever) you see

a white rectangle , replace it with a .

- (b) Calculate the dimension of the Checkered Flag. (Once again, the version you created above is actually the negative version of the actual Checkered Flag fractal, which would be black where yours is white and vice versa.)

2. For each of the following seeds  $s$ ,

- (i) Find the first 5 terms of the Mandelbrot sequence with seed  $s$  (find more, if you need to, until you can tell what's going on). (The Mandelbot sequence is just the list of results you get when you've started with a seed.)  
 (ii) Is this Mandelbrot sequence *escaping*, *periodic*, or *attracted*?  
 (iii) Will the point in the plane identified with the seed be a black point, or a non-black point?

- (a)  $s = 2$   
 (b)  $s = -2$   
 (c)  $s = -1$

3. *This won't be on the test, but just for completeness sake I will include these questions:* Evaluate the following, and then graph the results:

- (a)  $(2 - 3i)(2 + 3i)$   
 (b)  $3 - 5i + 7 - 11i$   
 (c)  $(4 + 7i) - (3 - 8i)$   
 (d)  $(6 + i)^2$

4. *This also won't be on the test, but if you understand complex numbers, this gives you a better feel for the Mandelbrot set* For each of the following seeds  $s$ ,

- (i) Find the first 5 terms of the Mandelbrot sequence with seed  $s$  (find more, if you need to, until you can tell what's going on). (The Mandelbot sequence is just the list of results you get when you've started with a seed.)  
 (ii) Is this Mandelbrot sequence *escaping*, *periodic*, or *attracted*?  
 (iii) Will the point in the plane identified with the seed be a black point, or a non-black point?

- (a)  $s = -i$
- (b)  $s = i/2$
- (c)  $s = 1 + i$