Math & Art — **Problem Set 2 (Group)** — Due 2/9/11, 9:30am

1. Consider a single house in Herculaneum, *The House of the Tuscan Colonnade*, also analyzed by the Watts. The floor plan is shown below.

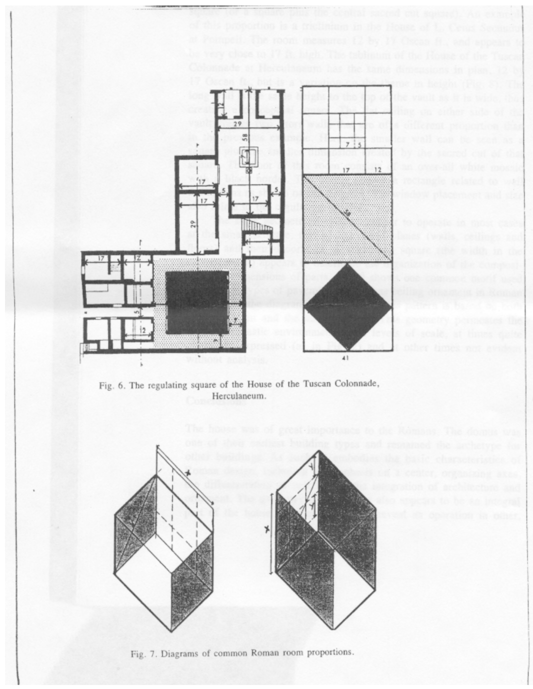

Fig. 6. The regulating square of the House of the Tuscan Colonnade, Herculaneum.

Fig. 7. Diagrams of common Roman room proportions.

The Watts measure the dimensions in Oscan feet [1] As you can see in the floor plan, the Watts found the following dimensions:

5	12	17	29	41

Find all ratios of these numbers that fall within the acceptance ranges of the Sacred Cut ratios (if any do). The acceptance ranges can be found under *Systems of Proportion - Acceptance ranges for the Sacred Cut* on the webpage I use for display in class, (listed as *Slides displayed in class* on OnCourse.)

[1] The Oscans were an early Italic people who built the original walls and towers of Pompeii, and may have founded Herculaneum.

2. Suppose you are going to be testing several works of art and architecture to see whether your favorite ratio appears. You are assuming that your measurements will be within 1% of the actual lengths, and so, as we found in class, the ratios will be off from the actual ratio of the lengths by (roughly) no more than 2%. Find the *acceptance range* for your ratio, if your favorite ratio is:

   (a) The Golden Ratio, $(1+\sqrt{5})/2$.

   (b) $(\sqrt{7}-\sqrt{5})/3$.

3. A Golden Ratio fanatic measures her box of Good Earth tea one morning and finds that the height of the box is 13 cm and the width is 7.9 cm. Is the ratio of the height to the width within the acceptance range for the Golden Ratio found in the previous problem?

4. Suppose you measure the length and width of a rectangle, paying attention to how accurately you are measuring. Your results:

$$\text{width } = 3 \text{ feet } \pm .02\% \qquad \text{length } = 6 \text{ feet } \pm .03\%.$$

   (a) Find the value of the ratio of the measured length to the measured width.

   (b) Find the range of values that the *actual* ratio of length to width could fall in. (That is, take into account the errors your measurements could have had.) *Remember:* the errors are percents, so you'll need to calculate what the actual error range is.

   (c) What's the furthest off from the measured ratio the *actual* ratio could be? Express your answer as a percent: *The actual ratio could be no more than ______% off from the measured ratio."*

5. The archaeologist in Example 1.4.1 decides that in fact, she is positive that her measurements of the wall mosaic were accurate to within $\pm 3\%$. That is, the wall mosaics measurements are:

$$\text{width} = 100\text{''} \pm 3\% \qquad \text{height} = 46\text{''} \pm 3\%.$$

With these new margins of error, could the mosaic be twice as wide as it is tall? (Even if the answer looks clear to you once you've done the calculations, please include a brief explanation of your conclusion.)

6. Rectangles in which the ratio of the longer side to the shorter side is the square root of an integer ($\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, etc) are called *root rectangles*. Some people feel that "they're harmoniously linked to each other and can have a powerful effect on viewers" [2]. The goal of this problem is to decide whether the Polyptych of St. Anthony by Piero della Francesca contains a $\sqrt{2}$ rectangle.

(a) What are the dimensions of the rectangle outlined in blue containing the three largest panels of the polyptych? (Just for consistency, please measure in centimeters, and measure very carefully.)

(b) Find the ratio of the long side to the short side.

(c) Assuming that your margin of error in each measurement is 1%, find an acceptance range for the "target ratio" $\sqrt{2}$.

(d) Based on your measurements of this reproduction, conclude whether these three panels may have been constructed to be a $\sqrt{2}$ rectangle.

[2] Michael S. Schneider, *Constructing the Universe - Dynamic Rectangles*

7. Suppose you have a line segment AB.

   (a) If you have a line segment AB, where on it must you put the point C so that the ratio $\frac{\overline{AB}}{\overline{AC}}$ is 2? When C is in this location, what is the value of the ratio $\frac{\overline{AC}}{\overline{BC}}$?

   (b) If you have a line segment AB, where must you put the point C so that the ratio $\frac{\overline{AB}}{\overline{AC}}$ is 1? When C is in this location, can you find the value of the ratio $\frac{\overline{AC}}{\overline{BC}}$? If not, why not?

   (c) Based on your responses to Exercises 7a and 7b, what are the smallest and largest possible values for the ratio $\frac{\text{length of the whole}}{\text{length of the greater}}$? *Even before calculating the Extreme and Mean ratio, this gives us a range that we know the Extreme and Mean ratio falls within.*

8. For each of the following, you'll be drawing a line that is cut in mean and extreme ratio (i.e. the Golden Ratio).

   (a) Suppose we want to draw a line cut in mean and extreme ratio, and we want the short segment to be 3 units long. How long should the long segment be? Draw such a line as carefully as possible.

   (b) Suppose we want to draw a line of length 6 that is cut in mean and extreme ratio. Where should we place the cut? Draw such a line, as carefully as possible.

   Look at the two lines with cuts that you've drawn. Do they look the same or different? (You don't necessarily have to address this in what you turn in, I just want you to pay attention to the big picture!)