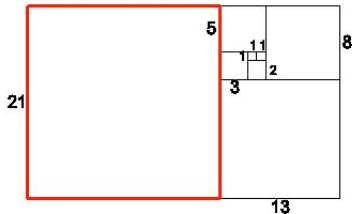# Fibonacci Numbers

The Fibonacci Sequence:

$\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots\}$

1. Begin with two 1's: $\{1, 1, \ldots$
2. Create the next term by adding the previous two
3. Return to Step 2

Notation:

F_N = the Nth term in the Fibonacci Sequence.

Example: $F_6 = 8$

$$F_{N+1} = \text{the term after the } N\text{th term in the Fibonacci Sequence}$$
$$F_N + 1 = \text{the } N\text{th term in the Fibonacci Sequence, plus 1.}$$

Formula for the Nth Fibonacci Number:

$$F_N = F_{N-1} + F_{N-2}$$

In Class Work

1. Given that $F_{19} = 4181$ and $F_{20} = 6765$, find

   (a) F_{21}

   (b) F_{18}

2. Given that $F_{31} = 1346269$ and $F_{33} = 3524578$, find

   (a) F_{32}

   (b) F_{34}

Solutions

1. Given that $F_{19} = 4181$ and $F_{20} = 6765$, find
   (a) F_{21}

   $$F_{21} = F_{20} + F_{19} = 6765 + 4181 = 10946$$

   (b) F_{18}

   $$F_{20} = F_{19} + F_{18} \Rightarrow F_{18} = F_{20} - F_{19} = 6765 - 4181 = 2584$$

2. Given that $F_{31} = 1346269$ and $F_{33} = 3524578$, find
   (a) F_{32}

   $$F_{33} = F_{32} + F_{31} \Rightarrow F_{32} = F_{33} - F_{31} = 3524578 - 1346269 = 2178309$$

   (b) F_{34}

   $$F_{34} = F_{33} + F_{32} = F_{33} + (F_{33} - F_{31}) = 3524578 + 2178309 = 5702887$$

In Class Work

Binet's Formula:

$$F_N = \frac{\left(\dfrac{1+\sqrt{5}}{2}\right)^N - \left(\dfrac{1-\sqrt{5}}{2}\right)^N}{\sqrt{5}} = \frac{\varphi^N - \left(\frac{1}{\varphi}\right)^N}{\sqrt{5}}$$

Verify that Binet's formula does indeed give the correct Fibonacci number, for $N = 4$ and $N = 7$, using your calculator.

Be careful with parentheses - you need a lot. On *my* calculator, I need to type:

$$\left(\left((1 + \sqrt{(5)})/2\right)^N - \left((1 - \sqrt{(5)})/2\right)^N \right) / \sqrt{(5)}$$