Recall: A line labeled as in Figure 1 is cut in Extreme and Mean ratio when

$$\frac{\overline{AB}}{\overline{AC}} = \frac{\overline{AC}}{\overline{CB}}.$$

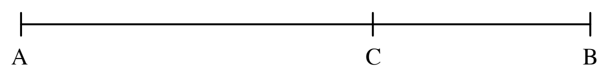

Figure 1:

When the above relationship is true, let
$\overline{CB} = 1$
$\overline{AC} = x$,
so $\overline{AB} = 1 + x$.

1. Rewrite the equation $\frac{\overline{AB}}{\overline{AC}} = \frac{\overline{AC}}{\overline{CB}}$ using the values shown above.

   Substituting 1 in for $\overline{CB}$, x in for $\overline{AC}$, and $1 + x$ in for $\overline{AB}$ results in the equation

$$\frac{1+x}{x} = \frac{x}{1}.$$

2. Solve the equation for x.

Hint: Quadratic formula: if $ax^2 + bx + c = 0$, then either $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ or $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$.

$$\begin{aligned}
\frac{1+x}{x} &= \frac{x}{1}. \\
\text{Cross-multiplying gives us} & \\
(1+x)(1) &= (x)(x) \\
\Rightarrow 1 + x &= x^2. \\
\text{Subtracting } 1 + x \text{ from both sides,} & \\
x^2 - x - 1 &= 0. \\
\text{Use the quadratic formula,} & \\
\text{with } a = 1, b = -1, c = -1 & \\
x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} \\
&= \frac{1 \pm \sqrt{1+4}}{2} \\
&= \frac{1 \pm \sqrt{5}}{2}
\end{aligned}$$

Since $\dfrac{1-\sqrt{5}}{2} < 0$, and since we know x is a length, $x = \dfrac{1+\sqrt{5}}{2}$.