Recall: Complex Numbers

Every point in the xy-plane can be written as a complex number:

$$(a, b) \leftrightarrow a + bi$$

Examples:

- $(5, 0) \leftrightarrow 5 + 0i = 5$
- $(0, -3) \leftrightarrow 0 - 3i = -3i$
- $(5, -3) \leftrightarrow 5 - 3i$

Recall: Creating the Mandelbrot Set - The Recursive Process

Start: Pick a point $s = (a, b)$ to be the seed.

Start: Write s as a complex number, $s = a + bi$

Next: Let $z_0 = 0 + 0i = 0 \leftrightarrow$ starting at point $(0, 0)$

1. $n = 0$
2. New n=old n+1
3. Let $z_n = z_{n-1}^2 + s$
4. Go to step 2

Form a list of the different values z_i takes on, beginning with 0:

$$\{z_0, z_1, z_2, z_3, \ldots\}$$

This list is called the Mandelbrot Sequence for the seed s.

Every seed (i.e. every point in the plane) has an associated Mandelbrot sequence

The Mandelbrot Set

- If the points in the Mandelbrot sequence for seed s move farther and farther away from the origin, without bound, we say the **seed s escapes**.
  - In that case, the seed is **not** in the Mandelbrot set.
  - Color the original point in space that corresponds to the seed a color. The color is determined by how fast the seed escapes.
- If the seed does not escape, whether its because the points in the Mandelbrot sequence approach some specific point, or they alternate between approaching a couple of points, then
  - the seed **is** in the Mandelbrot set
  - We make the original point **black**.

Mandelbrot Set - Example

Start: Pick the point (1,0). Write as a complex number:

seed$= s = 1 + 0i = 1$

Next: Form the equation $z_n = z_{n-1}^2 + 1$

Note: Don't change the "+1" part while working with this seed.

- $z_0 = 0$
- $n = 1$: $z_1^2 = z_0^2 + 1 = 0^2 + 1 = 1$
- $n = 2$: $z_2^2 = z_1^2 + 1 = 1^2 + 1 = 2$
- $n = 3$: $z_3^2 = z_2^2 + 1 = 2^2 + 1 = 5$
- $n = 4$: $z_4^2 = z_3^2 + 1 = 5^2 + 1 = 26$
- $n = 5$: $z_5^2 = z_4^2 + 1 = 26^2 + 1 = 677$
- etc

Mandelbrot sequence associated with $s = 1 + 0i$: $\{0, 1, 2, 5, 26, 677, \ldots\}$
This sequence will continue to grow without bound.
Thus the point $(1, 0)$ is **not** in the Mandelbrot set, and we will give it a color based on how fast it is **escaping**.

Examples We've Seen So Far:

- Pick $(1, 0) \Rightarrow s = 1 + 0i = 1$
  - Omitting initial 0, the Mandelbrot Sequence: $\{\underbrace{1}_{s}, 2, 5, 26, 677, \ldots\}$
  - The seed $(1, 0)$ **escapes**
  - $\therefore$ $(1, 0)$ is **not** in the Mandelbrot set. Color it some non-black color, based on how fast it escapes
- Pick $(0, 0) \Rightarrow s = 0 + 0i = 0$
  - Omitting initial 0, the Mandelbrot Sequence: $\{\underbrace{0}_{s}, 0, 0, 0, \ldots\}$
  - The seed $(0, 0)$ is attracted
  - $(0, 0)$ **is** in the Mandelbrot set. Color it black

Examples We've Seen So Far:

- Pick $(1, 0) \Rightarrow s = 1 + 0i = 1$
  - Omitting initial 0, the Mandelbrot Sequence: $\{1, 2, 5, 26, 677, \ldots\}$
  - The seed $(1, 0)$ **escapes**
  - $\therefore (1, 0)$ is **not** in the Mandelbrot set. Color it some non-black color, based on how fast it escapes
- Pick $(0, 0) \Rightarrow s = 0 + 0i = 0$
  - Omitting initial 0, the Mandelbrot Sequence: $\{0, 0, 0, 0, \ldots\}$
  - The seed $(0, 0)$ is attracted
  - $(0, 0)$ **is** in the Mandelbrot set. Color it black
- Pick $(-0.5, 0) \Rightarrow s = -0.5 + 0i$
  - Mandelbrot Seq: $\{-0.5, -0.25, -0.44, -0.31, -0.40, -0.34, \ldots\}$
  - The seed $(-0.5, 0)$ is attracted
  - $(-0.5, 0)$ **is** in the Mandelbrot set. Color it black
- $s = 0 + i$
  - Omitting 0, the Mandelbrot Seq: $\{i, -1 + i, -i, -1 + i, -i, -1 + i, \ldots\}$
  - The seed $(0, 1)$ is periodic
  - $(0, 1)$ **is** in the Mandelbrot set. Color it black