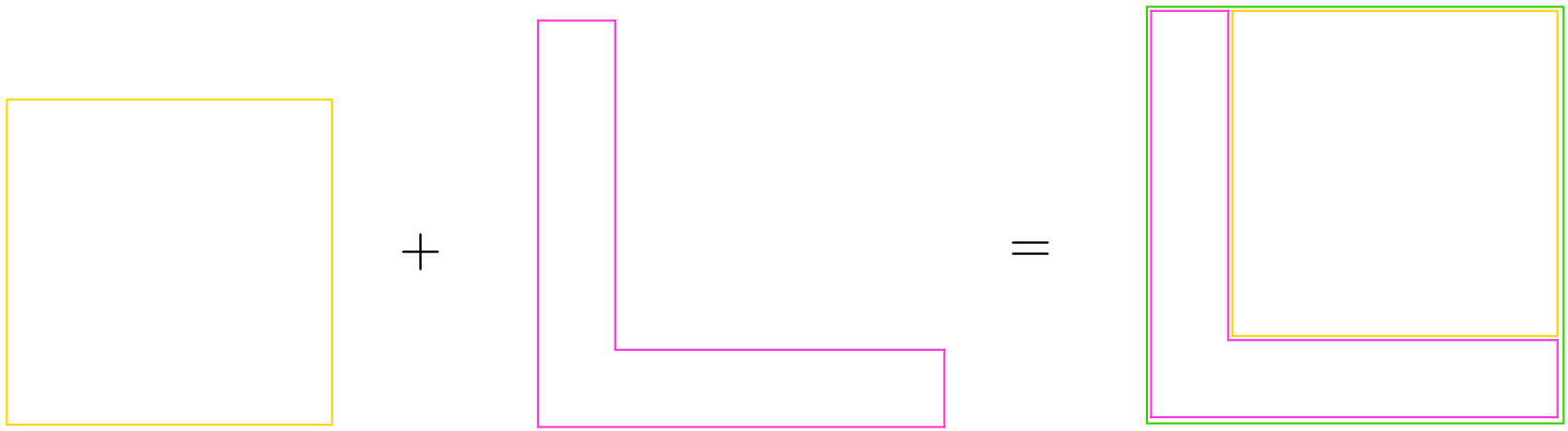
Recall - Gnomons

- **Definition:** A shape is **gnomon** an original figure if you can attach the shape to the original figure to create a new figure that is geometrically similar to the original figure.



Original figure + Gnomon = New figure similar to original

In Class Group Work

1. Sketch a figure that is gnomon to a rectangle of length 2 and width 1; label any sides with appropriate lengths.

2. Sketch a figure that is gnomon to a circle of radius 3; label any relevant portions with appropriate lengths.

3. Do any rectangles have square gnomons?

   If so, which? That is, what can you say about a rectangle that has a square gnomon?

   Consider:

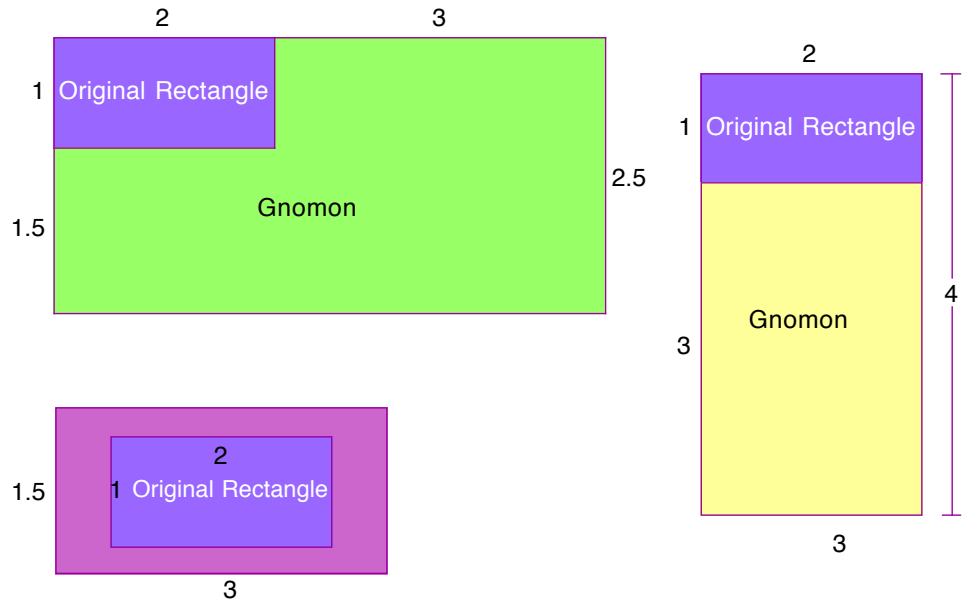
   - Rectangle has square gnomon if rectangle+square is **similar** to rectangle.
   - Draw a couple different shaped rectangles
   - For each sketched rectangle, which side would you attach square to for new rectangle to have any chance of being gnomon to original?
   - Can you set up any proportions?

Solutions

1. Describe a figure that is gnomon to a rectangle of length 2 and width 1.

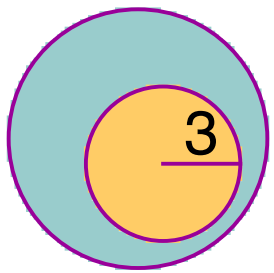
   Attach some shape to the 2×1 rectangle so that the end result is a rectangle similar to the 2×1 rectangle.

   Three possibilities:



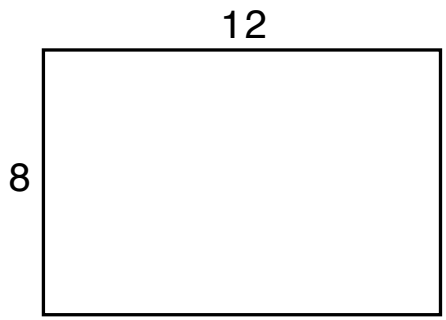
Solutions

2. Describe a figure that is gnomon to a circle of radius 3.
   Any ring with inner radius 3 and larger outer radius will do:



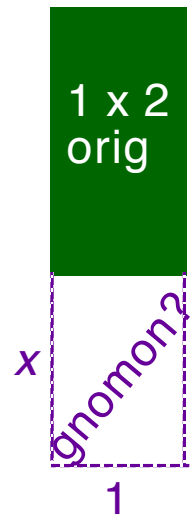
Rectangles with Rectangular or Square Gnomons

If we have a rectangle that is 1×2:



...what can we say about a rectangular or square gnomon?

Can it attach to the short side?



Doesn't work! $\frac{2+x}{2} \overset{\text{must}}{=} \frac{1}{1}$.
Impossible

Rectangles with Rectangular or Square Gnomons

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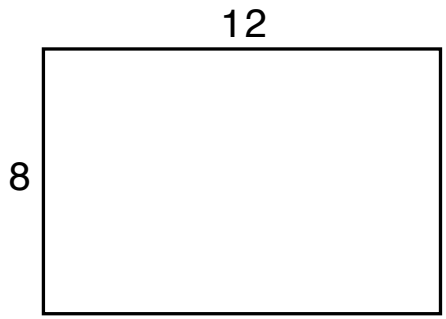

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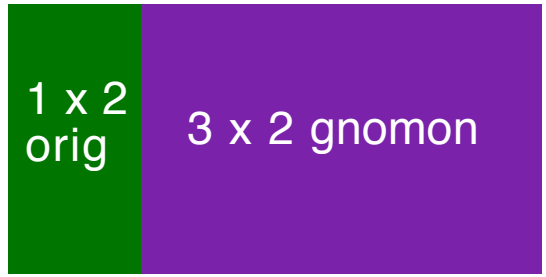
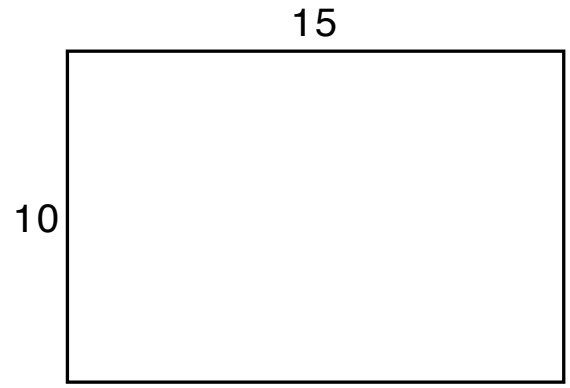


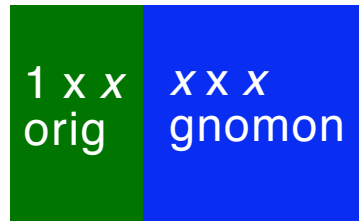
Must attach to long side!

Rectangles with Square Gnomons

• Start with a rectangle that is $1 \times x$ (where x is the long side.)

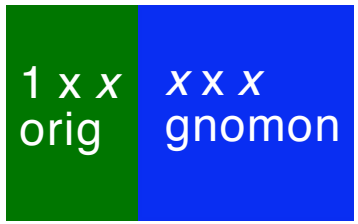


As we just saw, in order to have a **square** gnomon, the gnomon must attach to the long side – the side of length x.



Thus the square gnomon must have side x.

Rectangles with Square Gnomons



In order for the square to be gnomon to the original rectangle, the new rectangle must be similar to the original. Thus the ratio of the long sides must equal the ratio of the short sides:

$$\frac{x}{1} = \frac{1+x}{x} \quad \Rightarrow \quad x^2 - x - x = 0$$

We've seen this quadratic equation before, and we know this means $x = \varphi$.

Thus in both the original and the new rectangles, the ratio $\dfrac{\text{long side}}{\text{short side}} = \varphi$.