Review: Error Ranges

Example 1: Suppose $L_m = 40 \pm 2\text{m}$, $W_m = 6 \pm 0.5\text{m}$.
The error ranges for these measurements are

$$\begin{aligned} L_m = 40m &\quad\Rightarrow\quad 38m \le L_a \le 42m \\ W_a = 6m &\quad\Rightarrow\quad 5.5m \le W_a \le 6.5m \end{aligned}$$

Thus the error range for the measured ratio $\frac{L_m}{W_m}$ is

$$\begin{aligned} \frac{L_m}{W_m} = \frac{40}{6} &\quad\Rightarrow\quad \frac{\text{smallest } L}{\text{biggest } W} \le \frac{L_a}{W_a} \le \frac{\text{largest } L}{\text{smallest } W} \\ &\quad\Rightarrow\quad \frac{38}{6.5} \le \frac{L_a}{W_a} \le \frac{42}{5.5} \\ &\quad\Rightarrow\quad 5.85 \le \frac{L_a}{W_a} \le 7.36 \end{aligned}$$

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Example 2: (Same, except use percentage margins of error)

Suppose $L_m = 40$m and $W_m = 6$m, w/in 1%.

The error ranges for these measurements are

$$L_m = 40m \quad \Rightarrow \quad 0.99 \times 40m \leq L_a \leq 1.01 \times 40m \Rightarrow 39.6m \leq L_a \leq 40.4$$
$$W_a = 6m \quad \Rightarrow \quad 0.99 \times 6m \leq W_a \leq 1.01 \times 6m \Rightarrow 5.4m \leq W_a \leq 6.6$$

Saw last time: if both margins of error in measurement are 1% then margin of error for ratio is (roughly) 2%.

Thus since $\frac{L_m}{W_m} = \frac{40}{6}$, the error range for the measured ratio $\frac{L_m}{W_m}$ is

$$\frac{L_m}{W_m} = \frac{40}{6} \quad \Rightarrow \quad 0.98 \times \frac{40}{6} \leq \frac{L_a}{W_a} \leq 1.02 \times \frac{40}{6}$$
$$\Rightarrow \quad 6.53 \leq \frac{L_a}{W_a} \leq 6.8$$

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Using Error Ranges:
In both examples, $L_m = 40$m and $W_m = 6$m. $\frac{40}{6}$ is near 7. Is it near enough to pursue the idea that perhaps the ruin was intentionally built in a 7:1 ratio?

- If the error ranges for L and W are as in Example 1, $5.85 \leq \frac{L_a}{W_a} \leq 7.36$
- If the error ranges for L and W are as in Example 2, $6.53 \leq \frac{L_a}{W_a} \leq 6.8$

Thus:
In Example 2, 7 *is* in error range, so Accept that ruin *might have been* intentionally built in 7:1 ratio. Pursue further.
In Example 1, 7 not in error range, so reject that ruin was intentionally built in 7:1 ratio

Where we're headed:

- We've looked at **error ranges** – the interval around a measured ratio into which the actual ratio might fall.
- If all we're doing is taking ratios, rather than checking whether something uses a system of proportions, OR if we're only interested in a few ratios, then this method is fine.
- But if we're interested in whether a lot of measured ratios are sufficiently close to one or more ratios in a system of proportions, then it would be faster to have an **acceptance range** around the numbers in the system of proportions that we can check our measured ratios against.
- In other words, reversing the procedure.
- That is where we're heading next, using the Garden Houses of Ostia as our motivation.