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February 9, 2008

Math 102 Students
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Dear Calculus Students:

My boss, Mr. Burns, has recently decided that the time is ripe to diversify. Sector 7G of his nuclear power plant consistently under-performs, and Mr. Burns has been investigating ways to convert that sector to some other use. An employee there has accidentally discovered a possibility: through an odd series of events, this employee discovered that donuts stored in Sector 7G last considerably longer than the usual donut. Mr. Burns has asked me to look into converting that portion of the power plant to a donut factory.

I've done some research and have a rough idea about the potential costs and revenues, but now Mr. Burns tells me an emergency has come up and I must turn my attention elsewhere. Needless to say, he still wants to me to complete my research into the conversion from nuclear power plant to donut factory, so I'm turning to you to help me finish up the calculations. I'm hoping that if I tell you the results of my research so far, you can answer some specific questions, and provide some analysis.

The plan is to distribute the donuts to retailers in cases of 100.

The fixed costs of converting Sector 7G will be approximately \$7500. (We can keep this cost low because the employee who initially discovered the benefits of radiation on donuts has volunteered to do the work himself at a low cost in exchange for lifetime employment in the donut factory.)

Labor costs per case of donuts will initially decrease as we gain more efficiency, but these costs will increase if the number of donuts we make becomes sufficiently high that we have to pay overtime.

As for material costs, they too will decrease if we make enough to buy in bulk, but if we produce enough, we'll max out the capacity of our bulk suppliers in Springfield, and costs will rise when we have to ship in materials from Capitol City.

Here are the results of my extensive research into the materials costs, the labor costs, and the wholesale price the market will bear for several different levels of production.

Cases Produced	500	1000	2000	3000	4000	5000	6000	7000
Materials cost per case (\$)	900	625	225	30	40	250	625	1225
Labor cost per case (\$)	225	125	20	50	225	525	950	1525
Wholesale price per case (\$)	60	85	125	165	185	200	200	185

There are several vital pieces of information Mr. Burns has asked for. Specifically, he would like to know the number of cases of donuts we must produce to break even, the number we must produce to maximize our total profit, and the number we must produce to maximize the profit per case. He would, of course, also like to know what the profits would be in each situation.

I am quite interested in your insightful analysis of this conversion. Please do share with me the reasoning behind your conclusions, so that I can better pass the information onto Mr. Burns.

I will need your report by February 29 in order to prepare for production for next fall.

Sincerely,
Waylon Smithers

A Few Comments From Your Professor

There's no particular reason why the information requested should occur at one of the given data points. I would suggest that you try to find one function that models the materials cost, a different function that models the labor cost, and a third that models the wholesale price reasonably well. First plotting the data points that Mr. Smithers provided should give you an idea of the types of functions that will be likely candidates. Once you have these functions, you should be able to determine the necessary equations to find the requested information.

When you're writing your response to Mr. Smithers, you will of course need to be as professional as possible. Do not simply ask him to take you at your word – convince him of the validity of your conclusions by giving him a clear explanation. I expect you'll need to include graphs to help support your reasoning. For that reason, it makes sense for you to use Maple for your calculations rather than a graphing calculator.

A few Maple commands that might prove handy:

You can define a list of points in Maple as follows:

```
L:=[ [0,0], [1,1], [2,2], [3,3], [4,4] ];
```

(I just named it L for *list* – I've learned from experience that giving it a name that tells you what it is helps you keep track, so you may want to name your lists something like MC , LC and WP .)

You can that plot the list of points L , with each point connected to the next by a line, simply by typing

```
plot(L);
```

Graphing a list of points and a function on the same set of axes is a bit more complicated than graphing two functions on the same set of axes as you've already done. In order to graph a function $f(x)$ that you've already defined in Maple on the same set of axes as your list L , you need to do the following:

```
with(plots);  
p1:=plot(L);  
p2:=plot(f(x), x=a..b);
```

The *with(plots)* command loads a special graphing package. You only need to load it once – you can just type that at the beginning of your Maple worksheet if you want to. (You will need to enter it every time you open the Maple file, however).

Be Careful! Save frequently while you're working with Maple! E-mail yourself the Maple file. All of these public computers are automatically wiped clean every night, so you can't leave the file there for the next day.