In Class Work

Evaluate the following integrals:

1. $\int e^x \sqrt{e^x + 4}\, dx$
2. $\int_0^{\pi/4} \sec(x)\tan(x)\, dx$
3. $\int x^2 e^{x^3+5}\, dx$
4. $\int_1^2 x(3x+1)^5\, dx$
5. $\int \frac{\cos(1/x)}{x^2}\, dx$
6. $\int_0^1 x\sqrt{3-2x}\, dx$
7. $\int \frac{x^2 + 3\sqrt{x}}{\sqrt{x}}\, dx$
8. $\int_2^5 \frac{x+1}{\sqrt[3]{3x^2+6x+5}}\, dx$
9. $\int 2x \sec^2(1-x^2)\, dx$
10. $\int_0^1 \frac{x^3}{1+x^2}\, dx$
11. $\int \frac{e^{3x}}{(1+e^{3x})^5}\, dx$
12. $\int_0^{\pi/4} x\tan(x^2)\, dx$

Solutions

1. $\int e^x\sqrt{e^x+4}\,dx$

- Let $u =$ the inside of the composition $= e^x + 4$
  Then $du = e^x\,dx$
- Substitute:
$$\int e^x\sqrt{e^x+4}\,dx = \int \sqrt{u}\,du$$
- Integrate:
$$\int e^x\sqrt{e^x+4}\,dx = \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C$$
- Back-substitute:
$$\int e^x\sqrt{e^x+4}\,dx = \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(e^x+4)^{3/2} + C$$

Solutions

2. $\displaystyle\int_0^{\pi/4} \sec(x)\tan(x)\ dx$

Because $\dfrac{d}{dx}\big(\sec(x)\big) = \sec(x)\tan(x)$,

$$\int_0^{\pi/4} \sec(x)\tan(x)\ dx = \sec(x)\Big|_0^{\pi/4} = \sec(\pi/4) - \sec(0) = \sqrt{2} - 1$$

Solutions

3. $\int x^2 e^{x^3+5}\, dx$

- Let u = the inside of the composition $= x^3 + 5$
  Then $du = 3x^2\, dx \Rightarrow \frac{1}{3}\, du = x^2\, dx$
- Substitute:
$$\int x^2 e^{x^3+5}\, dx = \frac{1}{3}\int e^u\, du$$
- Integrate:
$$\int x^2 e^{x^3+5}\, dx = \frac{1}{3}\int e^u\, du = \frac{1}{3}e^u + C$$
- Back-substitute:
$$\int x^2 e^{x^3+5}\, dx = \frac{1}{3}\int e^u\, du = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3+5} + C$$

4. $\displaystyle\int_1^2 x(3x+1)^5\ dx$

Let $u = 3x + 1$. Then $du = 3\ dx$, so $\frac{1}{3}\ du = dx$.
Also, $u - 1 = 3x$, so $\frac{1}{3}(u-1) = x$.
And finally, when $x = 1$, $u = 4$, and when $x = 2$, $u = 7$.

$$\int_1^2 x(3x+1)^5\ dx = \frac{1}{3}\int_4^7 \frac{1}{3}(u-1)u^5\ du = \frac{1}{9}\int_4^7 u^6 - u^5\ du = \frac{1}{9}\left(\frac{1}{7}u^7 - \frac{1}{6}u^6\right)\Bigg|_4^7.$$

Finish by evaluating at 4 and 7.

5. $\displaystyle\int \frac{\cos(1/x)}{x^2}\,dx$

Let $u = \frac{1}{x}$. Then $du = -\frac{1}{x^2}\,dx$, so $-\,du = \frac{1}{x^2}\,dx$.

$$\int \frac{\cos(1/x)}{x^2}\,dx = -\int \cos(u)\,du = -\sin(u) + C = -\sin(1/x) + C$$

6. $\int_0^1 x\sqrt{3-2x}\,dx$

Let $u = 3-2x$. Then $du = -2\,dx$, so $-\frac{1}{2}\,du = dx$.
Also $x = 0 \Rightarrow u = 3$ and $x = 1 \Rightarrow u = 1$.
Finally, if $u = 3-2x$, then $u-3 = -2x$ so $\frac{1}{2}(3-u) = x$.

$$\begin{aligned}\int_0^1 x\sqrt{3-2x}\,dx &= -\frac{1}{2}\int_3^1 \frac{1}{2}(3-u)\sqrt{u}\,du = -\frac{1}{4}\int_3^1 3u^{1/2} - u^{3/2}\,du \\ &= -\frac{1}{4}\left(3\cdot\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right)\Big|_3^1 \\ &= -\frac{1}{4}\left[\left(2(1) - \frac{2}{5}(1)\right) - \left(2(3)^{3/2} - \frac{2}{5}(3)^{5/2}\right)\right]\end{aligned}$$