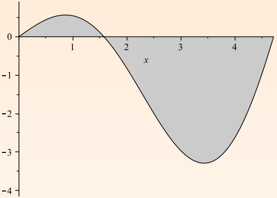
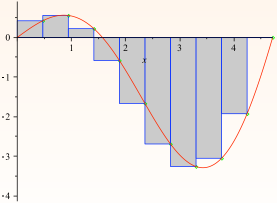
Signed Area - approximating with rectangles

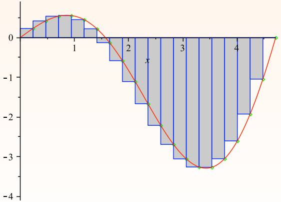
For $f(x) = x\cos(x)$, the signed area between the graph of $f(x)$ and the x-axis is shown below:



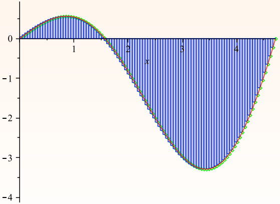
R_{10}



R_{20}

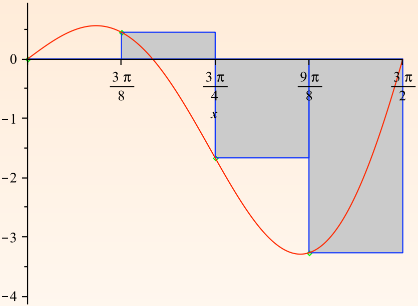


R_{100}



Recall: Left sum

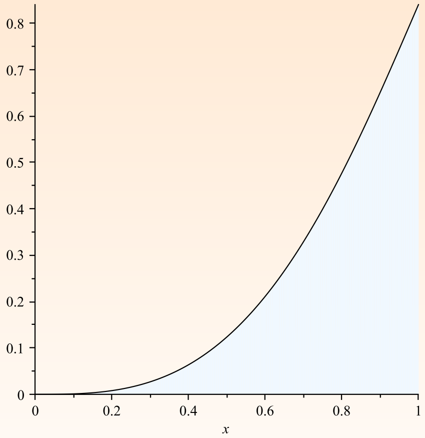
Let $f(x) = x\cos(x)$, and use 4 subintervals and a left sum to approximate the area between $f(x) = x\cos(x)$ and the x-axis from $x = 0$ to $x = \frac{3\pi}{8}$.



$$\int_0^{3\pi/2} x\cos(x)\,dx \approx L_4 = \text{sums of areas of rectangles}$$

$$= b_1h_1 + b_2h_2 + b_3h_3 + b_4h_4$$

$$= \frac{3\pi}{8}\left(f(0)) + f\left(\frac{3\pi}{8}\right) + f\left(\frac{6\pi}{8}\right) + f\left(\frac{9\pi}{8}\right)\right)$$



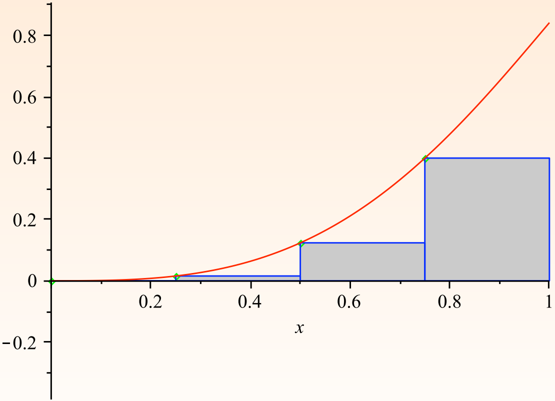
Let $I = \int_0^1 x \sin(x^2)\ dx$

1. Sketch L_4.
   Will L_4 overestimate or underestimate I?
2. Calculate L_4.
3. Sketch R_4.
   Will R_4 overestimate or underestimate I?
4. Calculate R_4.
5. Sketch M_4
6. Calculate M_4

Solutions

Let $I = \int_0^1 x \sin(x^2)\, dx$

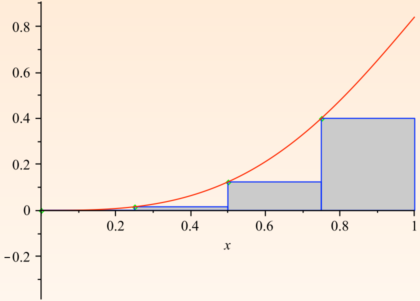
1. Sketch L_4. Does this overestimate or underestimate I?



L_4 will under-estimate, because f is increasing.

Solutions

2. Calculate L_4 by hand.



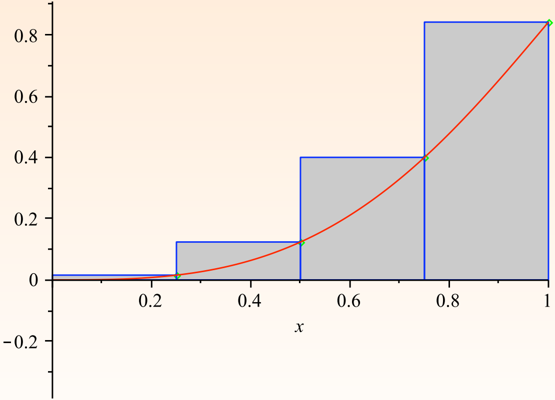
$$\begin{aligned}
\text{base} &= \Delta x = \frac{1-0}{4} = \frac{1}{4} \\
\text{height}_0 &= f(0) \\
\text{height}_1 &= f\left(1/4\right) \\
\text{height}_2 &= f\left(2/4\right) \\
\text{height}_3 &= f\left(3/4\right)
\end{aligned}$$

$$L_4 = \frac{1}{4}\left(f(0) + f\left(1/4\right) + f\left(2/4\right) + f\left(3/4\right)\right) = \frac{1}{4}\sum_{j=0}^{3} f\left(\frac{j}{4}\right)$$

Solutions

Let $I = \int_0^1 x \sin(x^2)\, dx$

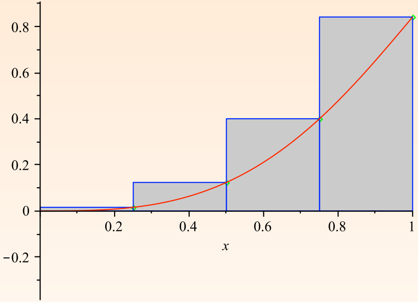
3. Sketch R_4. Does this overestimate or underestimate I?



R_4 will over-estimate, because f is increasing.

Solutions

4. Calculate R_4 by hand.

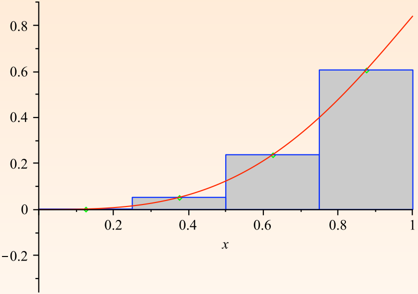


$$\begin{aligned} \text{base} &= \Delta x = \frac{1-0}{4} = \frac{1}{4} \\ \text{height}_1 &= f\left(1/4\right) \\ \text{height}_2 &= f\left(2/4\right) \\ \text{height}_3 &= f\left(3/4\right) \\ \text{height}_4 &= f(1) \end{aligned}$$

$$R_4 = \frac{1}{4}\left(f\left(1/4\right) + f\left(2/4\right) + f\left(3/4\right)\right) + f(1)\Bigg) = \frac{1}{4}\sum_{j=1}^{4} f\left(\frac{j}{4}\right)$$

Solutions

5, 6. Sketch and Calculate M_4.



$$\begin{aligned} \text{base} &= \Delta x = \frac{1-0}{4} = \frac{1}{4} \\ \text{height}_1 &= f(1/8) \\ \text{height}_2 &= f(3/8) \\ \text{height}_3 &= f(5/8) \\ \text{height}_4 &= f(7/8) \end{aligned}$$

$$R_4 = \frac{1}{4}\left(f(1/8) + f(3/8) + f(5/8)) + f(7/8)\right) = \frac{1}{4}\sum_{j=1}^{4} f\left(\frac{2j-1}{8}\right)$$