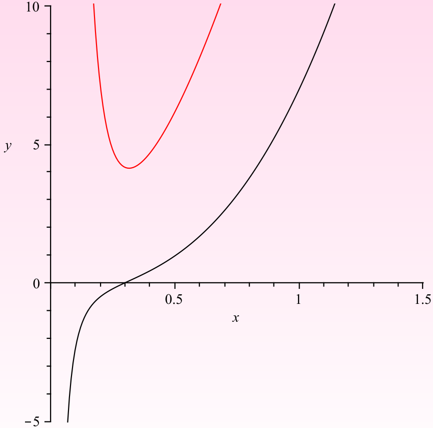
Summary: Derivatives Found Using Limit Definition

$$
\begin{aligned}
f(x) = 5 &\implies f'(x) = 0 \text{ (Fri 9/24)} \\
f(x) = 2x &\implies f'(x) = 2 \text{ (Fri 9/24)} \\
f(x) = x^2 &\implies f'(x) = 2x^1 \text{ (Fri 9/24)} \\
f(x) = x^3 &\implies f'(x) = 3x^2 \text{ (Fri 9/24)} \\
f(x) = x^{-1} &\implies f'(x) = -x^{-2} \text{ (Fri 9/24)} \\
f(x) = x^{1/2} &\implies f'(x) = \frac{1}{2}x^{-1/2} \text{ (Reading for Fri)}
\end{aligned}
$$

$$
\begin{aligned}
f(x) = x^2 - 3x &\implies f'(x) = 2x - 3 \text{ (Th 9/23)} \\
f(x) = 2x^2 - 3x &\implies f'(x) = 4x - 3 \text{ (RQ for Fri 9/24)} \\
f(x) = 3x^3 + 2x - 1 &\implies f'(x = 9x^2 + 2 \text{ (Reading for F)}
\end{aligned}
$$

Verifying a derivative

Graphs of $f(x) = 7x^e - \dfrac{1}{42x^2}$ and what we hope is in fact $f'(x)$:
$7ex^{e-1} + \frac{1}{21x^3}$



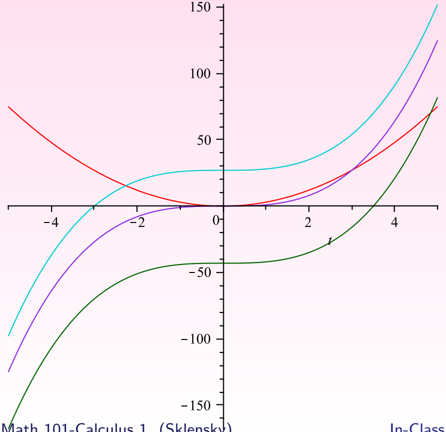
- $f(x)$ never flat
  $\Longrightarrow$ Slope $f'(x)$ never 0
- $f(x)$ always $\uparrow$
  $\Longrightarrow$ Slope $f'(x) > 0$
- $f(x)$ steep near $x = 0$
  $\Longrightarrow$ Slope $f'(x)$ large near 0
- $f(x)$ becomes less & less steep until $x =\sim 0.3$
  $\Longrightarrow$ Slope $f'(x) \downarrow$ to minimum at $x =\sim 0.3$.
- $f(x)$ then becomes steeper and steeper
  $\Longrightarrow$ Slope $f'(x) \uparrow$ again.

Verifying three antiderivatives

Original function: $v(t) = p'(t) = 3t^2$

Possible antiderivatives:

- $p(t) = t^3 + 27$
- $p(t) = t^3$
- $p(t) = t^3 - 43$



- $p'(t) = 0$ at $t = 0 \Longrightarrow$ $p(t)$ is flat at $t = 0$
- $p'(t) > 0$ for all $t \neq 0 \Longrightarrow$ $p(t) \uparrow$ for all $t \neq 0$
- $p'(t)$ first $\downarrow$ then $\uparrow \Longrightarrow$ $p(t)$ is getting flatter than steeper (since $p'(t) > 0$)

In Class Work

1. Find the derivatives of the following functions. If you have a graphing calculator, verify your answers by graphing f and f' on the same set of axes.

   (a) $f(x) = x^2 - 4x^{1/2} + 7$

   (b) $f(x) = x^3 - \frac{5}{x^2} + 2$

   (c) $f(x) = 2x^{\pi} + x^{-42} - 17x$

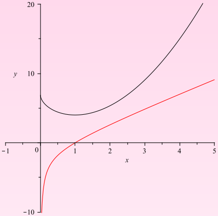
   (d) $f(x) = \frac{7}{x} - x + 4$

2. Find an *anti*derivative for each function in 1.

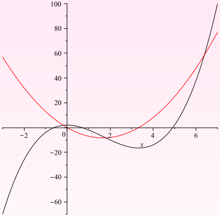
Solutions

1. Find the derivatives of the following functions.

(a) $f(x) = x^2 - 4x^{1/2} + 7$
$\Longrightarrow f'(x) = 2x - 2x^{-1/2}$



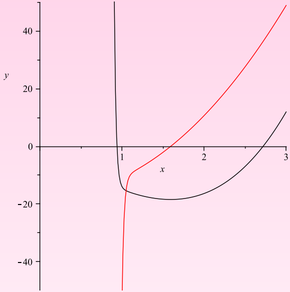
(b) $f(x) = x^3 - \dfrac{5}{x^2} + 2$
$\Longrightarrow f'(x) = 3x^2 + 10x^{-3}$



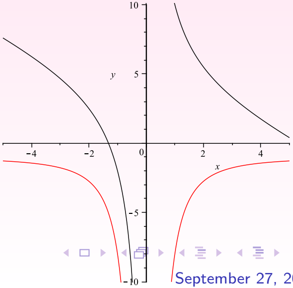
Solutions

1. Find the derivatives of the following functions.

(c) $f(x) = 2x^{\pi} + x^{-42} - 17x \implies$
$f'(x) = 2\pi x^{\pi-1} - 42x^{-43} - 17$



$f(x) = \dfrac{7}{x} - x + 4$
$\implies f'(x) = -7x^{-2} - 1$



Solutions

2. Find an *anti*derivative for each function in 1.

   (a) $f(x) = x^2 - 4x^{1/2} + 7 \Longrightarrow F(x) = x^3/3 - 4(\frac{2}{3})x^{3/2} + 7x$

   (b) $f(x) = x^3 - \dfrac{5}{x^2} + 2 \Longrightarrow F(x) = x^4/4 + 5/x + 2x$

   (c) $f(x) = 2x^{\pi} + x^{-42} - 17x \Longrightarrow$
   $F(x) = 2x^{\pi+1}/(\pi + 1) - x^{-41}/41 - 17x^2/2$

   (d) $f(x) = \dfrac{7}{x} - x + 4 \Longrightarrow F(x) =$ uh oh!

   When we try to follow our usual rule, it doesn't work!

   Does that mean that there *is* no function that antidifferentiates to $1/x$?

   Not necessarily – we've only looked at a very small collection of functions so far! But it does mean that no *power* function differentiates to be $1/x$!