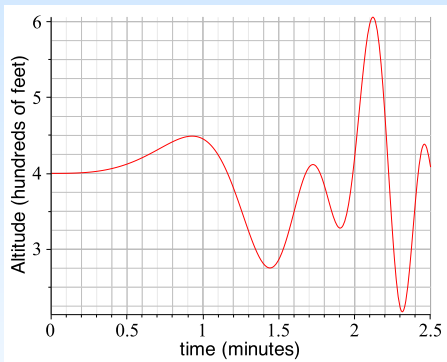


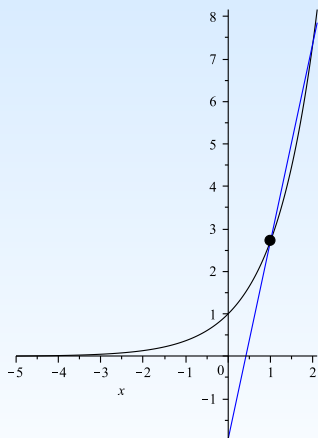
## In Class Work

The graph of the altitude  $A(t)$  of a hot air balloon after  $t$  minutes.



1. Is the balloon rising or falling at time  $t=2.4$ ?  $t=1.2$ ?
2. When is the balloon rising? falling?
3. When is the altitude function  $A(t)$  increasing? Decreasing?
4. Let  $V(t)$  denote the balloons upward velocity at time  $t$ . When is  $V$  positive? negative? zero?

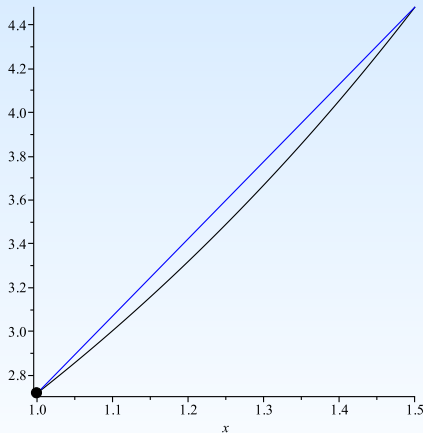
Goal: Estimate the rate  $f(t) = e^t$  changes at the point  $t = 1$ .



Average rate of change of  $f(t) = e^t$  from  $t = 1$  to  $t = 2$ :

$$\begin{aligned} \text{Average r.o.c.} &= \frac{f(2) - f(1)}{2 - 1} \\ &= \frac{e^2 - e^1}{1} \\ &\approx 4.67077 \end{aligned}$$

Goal: Estimate the rate  $f(t) = e^t$  changes at the point  $t = 1$ .



Average rate of change of  $f(t) = e^t$  from  $t = 1$  to  $t = 1.5$ :

$$\begin{aligned} \text{Average r.o.c.} &= \frac{f(1.5) - f(1)}{1.5 - 1} \\ &= \frac{e^{1.5} - e^1}{1} \\ &\approx 3.52681 \end{aligned}$$

**Goal: Estimate the rate  $f(t) = e^t$  changes at  $t = 1$ .**

From $t = 1$ to $t = \square$	Average rate of change
2	$\frac{e^2 - e^1}{2 - 1} \approx 4.67077$
1.5	$\frac{e^{1.5} - e^1}{1.5 - 1} \approx 3.52681$

**Goal: Estimate the rate  $f(t) = e^t$  changes at  $t = 1$ .**

From $t = 1$ to $t = \square$	Average rate of change
2	$\frac{e^2 - e^1}{2 - 1} \approx 4.67077$
1.5	$\frac{e^{1.5} - e^1}{1.5 - 1} \approx 3.52681$
1.1	$\frac{e^{1.1} - e^1}{1.1 - 1} \approx 2.85884$

**Goal: Estimate the rate  $f(t) = e^t$  changes at  $t = 1$ .**

From $t = 1$ to $t = \square$	Average rate of change
2	$\frac{e^2 - e^1}{2 - 1} \approx 4.67077$
1.5	$\frac{e^{1.5} - e^1}{1.5 - 1} \approx 3.52681$
1.1	$\frac{e^{1.1} - e^1}{1.1 - 1} \approx 2.85884$
1.01	$\frac{e^{1.01} - e^1}{1.01 - 1} \approx 2.73192$

**Goal: Estimate the rate  $f(t) = e^t$  changes at  $t = 1$ .**

From $t = 1$ to $t = \square$	Average rate of change
2	$\frac{e^2 - e^1}{2 - 1} \approx 4.67077$
1.5	$\frac{e^{1.5} - e^1}{1.5 - 1} \approx 3.52681$
1.1	$\frac{e^{1.1} - e^1}{1.1 - 1} \approx 2.85884$
1.01	$\frac{e^{1.01} - e^1}{1.01 - 1} \approx 2.73192$
1.001	$\frac{e^{1.001} - e^1}{1.001 - 1} \approx 2.71964$

**Goal: Estimate the rate  $f(t) = e^t$  changes at  $t = 1$ .**

From $t = 1$ to $t = \square$	Average rate of change
2	$\frac{e^2 - e^1}{2 - 1} \approx 4.67077$
1.5	$\frac{e^{1.5} - e^1}{1.5 - 1} \approx 3.52681$
1.1	$\frac{e^{1.1} - e^1}{1.1 - 1} \approx 2.85884$
1.01	$\frac{e^{1.01} - e^1}{1.01 - 1} \approx 2.73192$
1.001	$\frac{e^{1.001} - e^1}{1.001 - 1} \approx 2.71964$
1.0001	$\frac{e^{1.0001} - e^1}{1.0001 - 1} \approx 2.71842$



**Goal: Estimate the rate  $f(t) = e^t$  changes at  $t = 1$ .**

From $t = \square$ to $t = 1$	Average rate of change
0	$\frac{e^1 - e^0}{1 - 0} \approx 1.71828$
0.5	$\frac{e^1 - e^{0.5}}{1 - 0.5} \approx 2.13912$

**Goal: Estimate the rate  $f(t) = e^t$  changes at  $t = 1$ .**

From $t = \square$ to $t = 1$	Average rate of change
0	$\frac{e^1 - e^0}{1 - 0} \approx 1.71828$
0.5	$\frac{e^1 - e^{0.5}}{1 - 0.5} \approx 2.13912$
0.9	$\frac{e^1 - e^{0.9}}{1 - 0.9} \approx 2.58679$

**Goal: Estimate the rate  $f(t) = e^t$  changes at  $t = 1$ .**

From $t = \square$ to $t = 1$	Average rate of change
0	$\frac{e^1 - e^0}{1 - 0} \approx 1.71828$
0.5	$\frac{e^1 - e^{0.5}}{1 - 0.5} \approx 2.13912$
0.9	$\frac{e^1 - e^{0.9}}{1 - 0.9} \approx 2.58679$
0.99	$\frac{e^1 - e^{0.99}}{1 - 0.99} \approx 2.70474$

**Goal: Estimate the rate  $f(t) = e^t$  changes at  $t = 1$ .**

From $t = \square$ to $t = 1$	Average rate of change
0	$\frac{e^1 - e^0}{1 - 0} \approx 1.71828$
0.5	$\frac{e^1 - e^{0.5}}{1 - 0.5} \approx 2.13912$
0.9	$\frac{e^1 - e^{0.9}}{1 - 0.9} \approx 2.58679$
0.99	$\frac{e^1 - e^{0.99}}{1 - 0.99} \approx 2.70474$
0.999	$\frac{e^1 - e^{0.999}}{1 - 0.999} \approx 2.71692$

**Goal: Estimate the rate  $f(t) = e^t$  changes at  $t = 1$ .**

From $t = \square$ to $t = 1$	Average rate of change
0	$\frac{e^1 - e^0}{1 - 0} \approx 1.71828$
0.5	$\frac{e^1 - e^{0.5}}{1 - 0.5} \approx 2.13912$
0.9	$\frac{e^1 - e^{0.9}}{1 - 0.9} \approx 2.58679$
0.99	$\frac{e^1 - e^{0.99}}{1 - 0.99} \approx 2.70474$
0.999	$\frac{e^1 - e^{0.999}}{1 - 0.999} \approx 2.71692$
0.9999	$\frac{e^1 - e^{0.9999}}{1 - 0.9999} \approx 2.71814$

# Goal: Estimate the rate $f(t) = e^t$ changes at $t = 1$

To summarize:

$x = 1$ to $x = ?$	Average rate of change	$x = ?$ to $x = 1$	Average rate of change
2	$\frac{e^2 - e^1}{2 - 1} \approx 4.6708$	0	$\frac{e^1 - e^0}{1 - 0} \approx 1.7183$
1.5	$\frac{e^{1.5} - e^1}{1.5 - 1} \approx 3.52681$	0.5	$\frac{e^1 - e^{0.5}}{1 - 0.5} \approx 2.13912$
1.1	$\frac{e^{1.1} - e^1}{1.1 - 1} \approx 2.8588$	0.9	$\frac{e^1 - e^{0.9}}{1 - 0.9} \approx 2.5868$
1.01	$\frac{e^{1.01} - e^1}{1.01 - 1} \approx 2.7319$	0.99	$\frac{e^1 - e^{0.99}}{1 - 0.99} \approx 2.7047$
1.001	$\frac{e^{1.001} - e^1}{1.001 - 1} \approx 2.7196$	0.999	$\frac{e^1 - e^{0.999}}{1 - 0.999} \approx 2.7169$
1.0001	$\frac{e^{1.0001} - e^1}{1.0001 - 1} \approx 2.7184$	0.9999	$\frac{e^1 - e^{0.9999}}{1 - 0.9999} \approx 2.7181$

# Goal: Estimate the slope of the line tangent to

$$f(t) = e^t \text{ at } t = 1.$$

$t = 1$	Slope from $(1, e^1)$	$t = ?$	Slope from $(t, e^t)$
2	$\frac{e^2 - e^1}{2 - 1} \approx 4.6708$	0	$\frac{e^1 - e^0}{1 - 0} \approx 1.7183$
1.5	$\frac{e^{1.5} - e^1}{1.5 - 1} \approx 3.52681$	0.5	$\frac{e^1 - e^{0.5}}{1 - 0.5} \approx 2.13912$
1.1	$\frac{e^{1.1} - e^1}{1.1 - 1} \approx 2.8588$	0.9	$\frac{e^1 - e^{0.9}}{1 - 0.9} \approx 2.5868$
1.01	$\frac{e^{1.01} - e^1}{1.01 - 1} \approx 2.7319$	0.99	$\frac{e^1 - e^{0.99}}{1 - 0.99} \approx 2.7047$
1.001	$\frac{e^{1.001} - e^1}{1.001 - 1} \approx 2.7196$	0.999	$\frac{e^1 - e^{0.999}}{1 - 0.999} \approx 2.7169$
1.0001	$\frac{e^{1.0001} - e^1}{1.0001 - 1} \approx 2.7184$	0.9999	$\frac{e^1 - e^{0.9999}}{1 - 0.9999} \approx 2.7181$

## Reading Question 1(b)

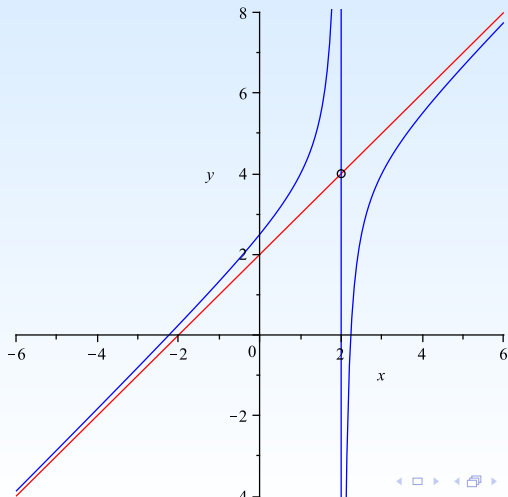
$$f(x) = x^2 + 1$$

$x = 2$ to $x = ?$	Slope from $(2, 5)$ to $(x, x^2 + 1)$	$x = ?$ to $x = 2$	Slope from $(x, x^2 + 1)$ to $(2, 5)$
3	$\frac{f(3) - 5}{3 - 2} \approx 5$	1	$\frac{5 - f(1)}{2 - 1} \approx 3$
2.1	$\frac{f(2.1) - 5}{2.1 - 2} \approx 4.1$	1.9	$\frac{5 - f(1.9)}{2 - 1.9} \approx 3.9$
2.01	$\frac{f(2.01) - 5}{2.01 - 2} \approx 4.01$	1.99	$\frac{5 - f(1.99)}{2 - 1.99} \approx 3.99$
2.001	$\frac{f(2.001) - 5}{2.001 - 2} \approx 4.001$	1.999	$\frac{5 - f(1.999)}{2 - 1.999} \approx 3.999$
2.0001	$\frac{f(2.0001) - 5}{2.0001 - 2} \approx 4.0001$	1.9999	$\frac{5 - f(1.9999)}{2 - 1.9999} \approx 3.9999$



# For Reading Question #1

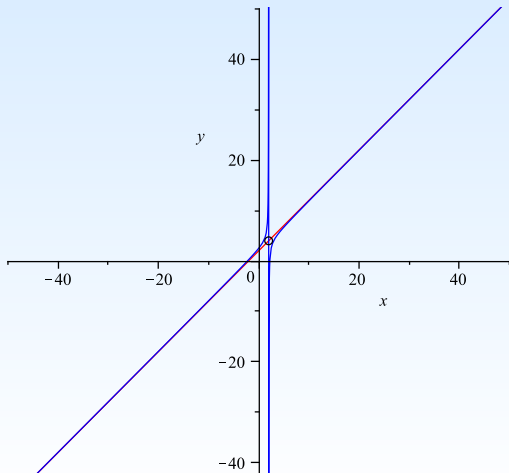
$$\frac{x^2 - 4}{x - 2}, \frac{x^2 - 5}{x - 2}$$



# For Reading Question #1

Zooming out:

$$\frac{x^2 - 4}{x - 2}, \frac{x^2 - 5}{x - 2}$$



# For Reading Question #1

Zooming back in:

$$\frac{x^2 - 4}{x - 2}, \frac{x^2 - 5}{x - 2}$$

