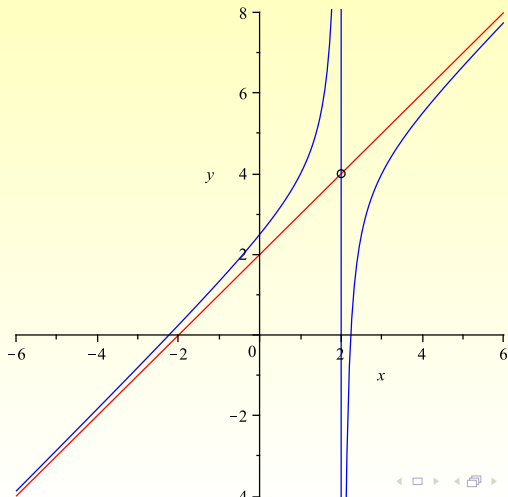


## For Reading Question #1 from Monday:

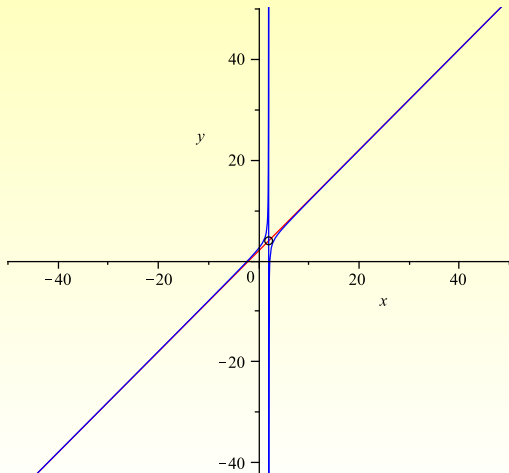
$$\frac{x^2 - 4}{x - 2}, \frac{x^2 - 5}{x - 2}$$



# For Reading Question #1 from Monday:

Zooming out:

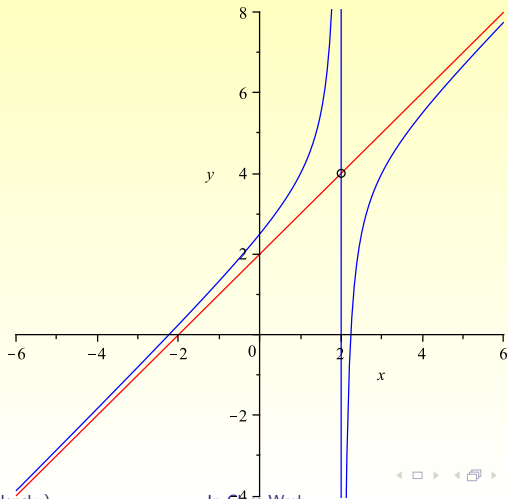
$$\frac{x^2 - 4}{x - 2}, \frac{x^2 - 5}{x - 2}$$



# For Reading Question #1 from Monday:

Zooming back in:

$$\frac{x^2 - 4}{x - 2}, \frac{x^2 - 5}{x - 2}$$



## For RQ # 2 from Monday: Example 1.2.2, from text

Evaluate  $\lim_{x \rightarrow -3} \frac{3x + 9}{x^2 - 9}$ .

$$\begin{aligned}\lim_{x \rightarrow -3^-} \frac{3x + 9}{x^2 - 9} &= \lim_{x \rightarrow -3^-} \frac{3(x + 3)}{(x + 3)(x - 3)} \\ &= \lim_{x \rightarrow -3^-} \frac{3}{x - 3} = -\frac{1}{2}\end{aligned}$$

Cancel factors of  $(x + 3)$ .

## For RQ # 2 from Monday: Example 1.2.2, from text

Evaluate  $\lim_{x \rightarrow -3} \frac{3x + 9}{x^2 - 9}$ .

$$\begin{aligned}\lim_{x \rightarrow -3^-} \frac{3x + 9}{x^2 - 9} &= \lim_{x \rightarrow -3^-} \frac{3(x + 3)}{(x + 3)(x - 3)} && \text{Cancel factors of } (x + 3). \\ &= \lim_{x \rightarrow -3^-} \frac{3}{x - 3} = -\frac{1}{2}\end{aligned}$$

**In the limit**, the cancellation is legal, because  $x$  is never **equal to**  $-3$ .

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**In the limit**, the cancellation is legal, because  $x$  is never **equal to**  $-3$ .

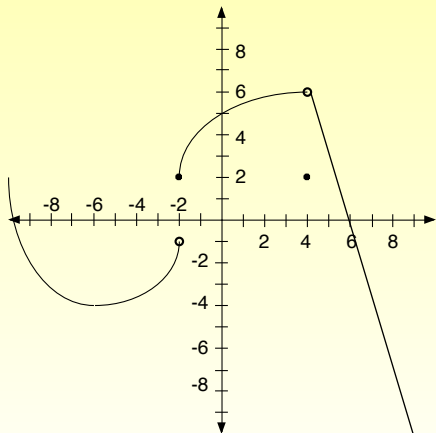
However, it is **not correct** to write

$$\frac{3(x + 3)}{(x + 3)(x - 3)} = \frac{3}{x - 3}$$

without some sort of note like **when  $x \neq -3$**

## In Class Work

Consider the function  $f(x)$  defined by the graph below. Find the following:



1.  $f(4)$

2.  $\lim_{x \rightarrow 4^+} f(x)$  and  $\lim_{x \rightarrow 4^-} f(x)$

3.  $\lim_{x \rightarrow 4} f(x)$

4. Is  $f$  continuous at  $x = 4$ ?

5.  $f(-2)$

6.  $\lim_{x \rightarrow -2^+} f(x)$  and  $\lim_{x \rightarrow -2^-} f(x)$

7.  $\lim_{x \rightarrow -2} f(x)$

8. Is  $f$  continuous at  $x = 2$ ?

9.  $f(-6)$

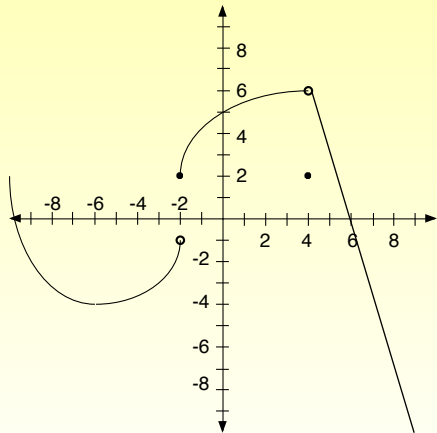
10.  $\lim_{x \rightarrow -6^+} f(x)$  and  $\lim_{x \rightarrow -6^-} f(x)$

11.  $\lim_{x \rightarrow -6} f(x)$

12. Is  $f$  continuous at  $x = 6$ ?

# Solutions to In Class Work

Consider the function  $f(x)$  defined by the graph below. Find the following:



1.  $f(4) = y$ -value of solid circle = 2:

2.  $\lim_{x \rightarrow 4^+} f(x) = 6, \lim_{x \rightarrow 4^-} f(x) = 6.$

**Remember:** for limits, we do not pay any attention to what happens **at**  $x = 4$ .

3.  $\lim_{x \rightarrow 4} f(x) = 6$ , since both the left- and right-limits were 6.

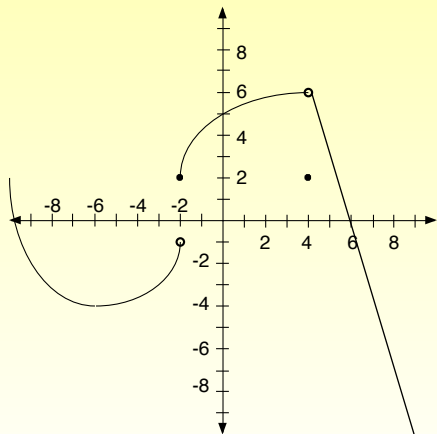
**Notice:**  $\lim_{x \rightarrow 4} f(x) \neq f(4).$

4. Can see  $f$  is not continuous at  $x = 4$ .



## Solutions to In Class Work

Consider the function  $f(x)$  defined by the graph below. Find the following:



5.  $f(-2) = y$ -value of closed circle = 2

6.  $\lim_{x \rightarrow -2^+} f(x) = 2,$   
 $\lim_{x \rightarrow -2^-} f(x) = -1$

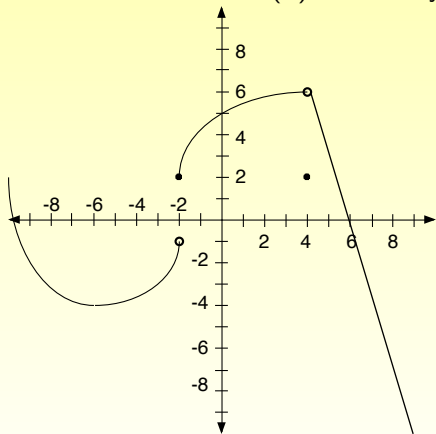
7.  $\lim_{x \rightarrow -2} f(x)$  d.n.e. (does not exist),  
because the left- and right- sided  
limits differ.

**Notice:** As with  $x = 4,$   
 $\lim_{x \rightarrow -2} f(x) \neq f(-2)$

8. Can see  $f$  isn't continuous at  
 $x = -2.$

# Solutions to In Class Work

Consider the function  $f(x)$  defined by the graph below. Find the following:



9.  $f(-6) = -4$

10.  $\lim_{x \rightarrow -6^+} f(x) = -4,$   
 $\lim_{x \rightarrow -6^-} f(x) = -4$

11.  $\lim_{x \rightarrow -6} f(x) = -4$

**Notice:**  $\lim_{x \rightarrow -6} f(x) = f(-6)$

12. Can see  $f$  is continuous at  $x = 6$

**ESTIMATE**  $\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x}$  by creating a table of values of  $f(x)$ :

From the left of  $x = 0$ :

$x$	$\frac{1 - \cos(x)}{x}$
-0.1	-0.049958
-0.01	-0.005000
-0.001	-0.000500
-0.0001	-0.000050
-0.00001	-0.000005

From the right of  $x = 0$ :

$x$	$\frac{1 - \cos(x)}{x}$
0.1	0.049958
0.01	0.005000
0.001	0.000500
0.0001	0.000050
0.00001	0.000005