Recall: Slope of Secant Line =Ave. Rate of Change

If we are looking at the graph of $y = f(x)$, then

$$\text{slope of secant line from } x = a \text{ to } x = b = \frac{\text{rise}}{\text{run}}$$
$$= \frac{\Delta y}{\Delta x}$$
$$= \boxed{\frac{f(b) - f(a)}{b - a}}$$

If we are looking at an object whose position is given by $f(t)$, then

$$\text{average r.o.c. from } t = a \text{ to } t = b = \frac{\text{change in position}}{\text{time}}$$
$$= \frac{f(t_{\text{final}}) - f(t_{\text{initial}})}{t_{\text{Final}} - t_{\text{initial}}}$$
$$= \boxed{\frac{f(b) - f(a)}{b - a}}.$$

What We're Really Interested In:

- **Slope of a Curve** (i.e, slope of the line tangent to the curve)

- **Instantaneous Rate of Change**

What We're Really Interested In:

- **Slope of a Curve** (i.e, slope of the line tangent to the curve)

  The closer b is to a, the closer the slope of the secant line is to the slope of the curve itself at $x = a$.

$$\text{slope of tangent line at } x = a = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

- **Instantaneous Rate of Change**

  The smaller the time interval, the closer the average rate of change is to the instantaneous rate of change at $t = a$.

$$\text{inst r.o.c. at } t = a \text{ is } \lim_{t \to a} \frac{f(t) - f(a)}{t - a}$$

$\therefore$ **Slope of Curve at $x = a$ = Inst. R. of C. at $x = a$**

In Class Work

1. Use the limit definition to find the slope of the line tangent to $f(x) = x^2 - 3x$ at the following points.

   If you happen to have learned a short cut for this process in a past class, do NOT use it here. The point of this work is to practice with and understand the limit definition of the slope of the secant line.

   (a) $x = 0$

   (b) $x = -2$

   (c) An unspecified point x

2. Find the equation of the line tangent to $f(x) = x^2 - 3x$ at $x = -2$.

Solutions

1. Use the limit definition to find the slope of the line tangent to $f(x) = x^2 - 3x$ at:

   (a) $x = 0$

$$
\begin{aligned}
m_{tan} &= \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} \\
&= \lim_{h \to 0} \frac{\left[(h)^2 - 3(h)\right] - \left[0^2 - 3(0)\right]}{h} \\
&= \lim_{h \to 0} \frac{h^2 - 3h}{h} \\
&= \lim_{h \to 0} h - 3 \\
&= -3
\end{aligned}
$$

Solutions

1. Use the limit definition to find the slope of the line tangent to $f(x) = x^2 - 3x$ at the following points.

   (b) $x = -2$

$$
\begin{aligned}
m_{tan} &= \lim_{h\to 0} \frac{f(-2+h) - f(-2)}{h} \\
&= \lim_{h\to 0} \frac{\Big[(-2+h)^2 - 3(-2+h)\Big] - \Big[2^2 - 3(-2)\Big]}{h} \\
&= \lim_{h\to 0} \frac{\big(4 - 4h + h^2 + 6 - 3h\big) - \big(4 + 6\big)}{h} \\
&= \lim_{h\to 0} \frac{-4h + h^2 - 3h}{h} = \lim_{h\to 0} -4 + h - 3 \\
&= -7
\end{aligned}
$$

Solutions

1. Use the limit definition to find the slope of the line tangent to $f(x) = x^2 - 3x$ at:

    (c) An unspecified point x

$$
\begin{aligned}
m_{tan} &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\left[(x+h)^2 - 3(x+h)\right] - \left[x^2 - 3(x)\right]}{h} \\
&= \lim_{h\to 0} \frac{\left(x^2 + 2xh + h^2 - 3x - 3h\right) - \left(x^2 - 3x\right)}{h} \\
&= \lim_{h\to 0} \frac{2xh + h^2 - 3h}{h} \\
&= \lim_{h\to 0} 2x + h - 3 \\
&= 2x - 3
\end{aligned}
$$

Solutions

2. Find the equation of the line tangent to $f(x) = x^2 - 3x$ at $x = -2$.

   Need: point and a slope

   Slope: $m_{tan}(-2) = -7$

   Point: Use point of tangency, $\big(-2, f(-2)\big) = (-2, 4+6) = (-2, 10)$

   Thus the tangent line is given by

$$y - 10 = -7(x+2) \Longrightarrow y = -7x - 4.$$