Local Extrema:

- *Maximum*=A high point on a graph. Plural: *Maxima*
- *Minimum*=A low point on a graph. Plural: *Minima*
- *Extremum*= An extreme point on the graph – that is, either a maximum or a minimum. Plural: *Extrema*

- *Local* = in a neighborhood (just like in English!)
- A *local maximum* is as high or higher than any point nearby –a "peak". A *local minimum* is as low or lower than any point nearby – a "valley".

Inflection Points

- While extrema are where a curve changes *direction*, *inflection points* are where a curve changes from being bent upward to being bent downward, or vice versa.

Exponential Functions:

An **exponential function** has the form

$$f(x) = b^x$$

where b is a *fixed* **positive** number, called the **base.**

Exponential Functions:

An **exponential function** has the form

$$f(x) = b^x$$

where b is a *fixed* **positive** number, called the **base.**

Examples: $g(x) = e^x$, $h(x) = 3^x$, and $k(x) = 4^x$ are all exponential functions.

Remember: e is an irrational number that shows up in many different places; like π it shows up enough that we give it's own name.

$$e \approx 2.71828...$$

In Class Work

1. (a) Is π^x an exponential function?
   (b) Is x^2 an exponential function?
2. If $f(x) = b^x$, where b is any positive number,
   (a) what is $f(0)$? (Your answer may be in terms of b)
   (b) what is $f(1)$? (Your answer may be in terms of b)
3. By plotting a few points, sketch the graphs of
   (a) $y = 2^x$
   (b) $y = 3^x$
4. If $b > 0$, what is the domain of $f(x) = b^x$? How about the range?
   Remember:
   - the *domain* is the set of inputs – what x does it make sense to plug in;
   - the *range* is the set of outputs – what values of $f(x)$ will you get out, if you plug in all possible values of x?

Solutions - In Class Work

1. (a) Is π^x an exponential function?

   Yes: the base, π is a constant, while the exponent is a variable.

   (b) Is x^2 an exponential function?

   No: the base is a variable while the exponent is fixed. This is a *power function*.

2. If $f(x) = b^x$, where b is any positive number,

   (a) what is $f(0)$?

   $f(0) = b^0 = 1$.

   Conclusion: The point $(0, 1)$ is on *every* exponential function.

   (b) what is $f(1)$?

   $f(1) = b^1 = b$.

   Conclusion: The graph of b^x goes through the point $(1, b)$.

Solutions - In Class Work

3. By plotting a few points, sketch the graphs of
   (a) $y = 2^x$
   (b) $y = 3^x$

2^x

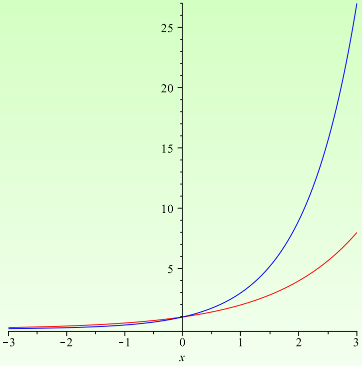
x	y
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$

3^x

x	y
-2	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
-1	$3^{-1} = \frac{1}{3^1} = \frac{1}{3}$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$

In Class Work

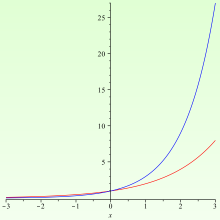
3. By plotting a few points, sketch the graphs of
   (a) $y = 2^x$
   (b) $y = 3^x$



Note: As long as $b > 1$, the graph of b^x will be increasing, and concave up.

In Class Work

4. If $b > 0$, what is the domain of $f(x) = b^x$? How about the range?
   Remember:
   - the *domain* is the set of inputs – what x does it make sense to plug in;
   - the *range* is the set of outputs – what values of $f(x)$ will you get out, if you plug in all possible values of x?



(a) As long as $b > 0$, b^x makes sense for all possible values of x, so the domain is all real numbers, or the interval $(-\infty, \infty)$.

(b) As long as $b > 0$, b^x will also be positive. We can get any value out, by choosing an appropriate value of x. Thus the range is all positive numbers, or $(0, \infty)$.

Recall:

A **logarithm function** with base b is denoted

$$f(x) = \log_b(x)$$

and is defined by $y = \log_b(x)$ iff $b^y = x$.

Inverse Functions

- Two functions are **inverses** of each other if
  - For all $x \in$ the domain of g, $f\big(g(x)\big) = x$
    and
  - For all $x \in$ the domain of f, $g\big(f(x)\big) = x$

Inverse Functions

- Two functions are **inverses** of each other if
  - For all $x \in$ the domain of g, $f\big(g(x)\big) = x$
    and
  - For all $x \in$ the domain of f, $g\big(f(x)\big) = x$
- **Notation:**
  If f and g are inverses of each other, we write $g(x) = f^{-1}(x)$.

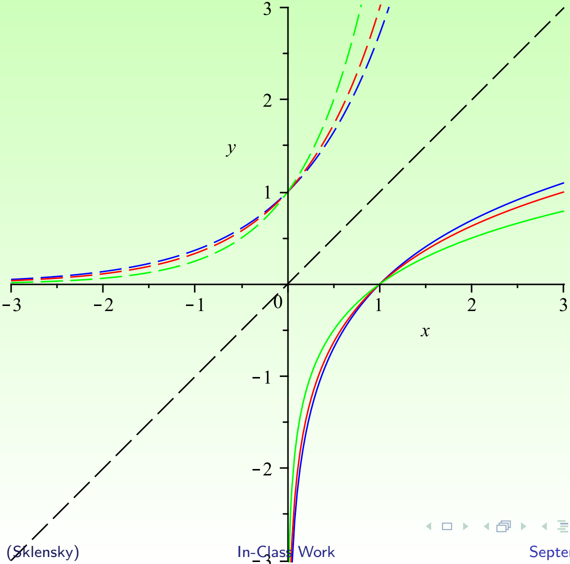
Inverse Functions

- Two functions are **inverses** of each other if
  - For all $x \in$ the domain of g, $f\big(g(x)\big) = x$
    and
  - For all $x \in$ the domain of f, $g\big(f(x)\big) = x$
- **Notation:**
  If f and g are inverses of each other, we write $g(x) = f^{-1}(x)$.
- **Examples of Inverse Functions**
  - $f(x) = x^2$ for $x \geq 0$; $f^{-1}(x) = \sqrt{x}$
  - $f(x) = 2x + 5$, $f^{-1}(x) = \frac{x-5}{2}$

More on Inverse Functions:

- If a point (a, b) is on the graph of $f(x)$, then that means that $f(a) = b$.
- In turn, that must mean that $f^{-1}(b) = f^{-1}(f(a)) = a$, so the point (b, a) is on the graph of $f^{-1}(x)$.
- This ends up meaning that the graph of f^{-1} is the same as the graph of f reflected across the line $y = x$.

Graphs of $\ln(x)$, $\log_3(x)$, and $\log_4(x)$, along with those of e^x, 3^x, and 4^x



Notice:

- $f(x) = \log_b(x)$ *always* passes through the points $(b, 1)$ and $(1, 0)$:

$$\square = \log_b(b) \Leftrightarrow b^{\square} = b \Leftrightarrow \square = 1$$
$$\text{Thus } (b, 1) \text{ is on the graph}$$
$$\square = \log_b(1) \Leftrightarrow b^{\square} = 1 \Leftrightarrow \square = 0$$
$$\text{Thus } (1, 0) \text{ is on the graph}$$

Notice:

- $f(x) = \log_b(x)$ *always* passes through the points $(b, 1)$ and $(1, 0)$:

$$\square = \log_b(b) \Leftrightarrow b^{\square} = b \Leftrightarrow \square = 1$$

Thus $(b, 1)$ is on the graph

$$\square = \log_b(1) \Leftrightarrow b^{\square} = 1 \Leftrightarrow \square = 0$$

Thus $(1, 0)$ is on the graph

- Logarithms are not defined at $x = 0$:

$$\square = \log_b(0) \Leftrightarrow b^{\square} = 0$$

Thus 0 is not in the domain of any logarithmic function

Also notice:

- Logarithms are not defined at negative numbers:

$$\square = \log_b(\text{negative number} \Leftrightarrow b^{\square} = \text{ negative number}$$

Since b is positive, no matter what power you raise it to, $b^{\square}$ will always be positive.