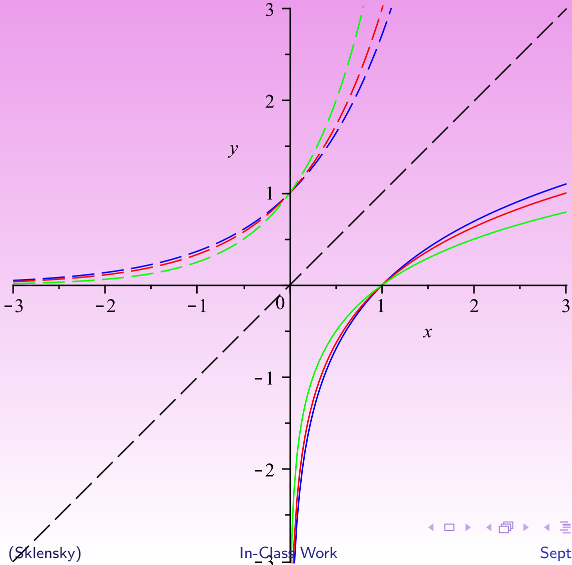
Graphs of $\ln(x)$, $\log_3(x)$, and $\log_4(x)$, along with those of e^x, 3^x, and 4^x



- The logarithm is defined in terms of exponential functions.
  $\log_b x$ is *defined to be* the power you raise b to, to get x.
  That is, if $x > 0$,
  $$y = \log_b x \Leftrightarrow b^y = x.$$

- **Compare that to:** squaring vs square roots.
  - The square root is defined in terms of squaring.
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  - The square root function is (more or less) the **inverse** of the squaring function: it *undoes* what the square does, and square undoes what the square root undoes.
  - That is,
    If $x \geq 0$, $\sqrt{(x^2)} = x$ and $(\sqrt{x})^2 = x$.
  - In other words:
    If $f(x) = \sqrt{x}$ and $g(x) = x^2$, then
    $$\text{if } x \geq 0, f\big(g(x)\big) = x \text{ and } g\big(f(x)\big) = x.$$

Inverse Functions

- Two functions are **inverses** of each other if
  - For all $x \in$ the domain of g, $f\big(g(x)\big) = x$
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- **Notation:**
  If f and g are inverses of each other, we write $g(x) = f^{-1}(x)$.
- **Examples of Inverse Functions**
  - $f(x) = x^2$ for $x \geq 0$; $f^{-1}(x) = \sqrt{x}$
  - $f(x) = 2x + 5$, $f^{-1}(x) = \frac{x-5}{2}$

More on Inverse Functions:

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- Taking f^{-1} of both sides of that equation, we get $f^{-1}(b) = f^{-1}\big(f(a)\big)$. Since f and f^{-1} undo each other, this means $f^{-1}(b) = a$.
- Thus the point (b, a) is on the graph of $f^{-1}(x)$.
- This ends up meaning that the graph of f^{-1} is the same as the graph of f reflected across the line $y = x$.

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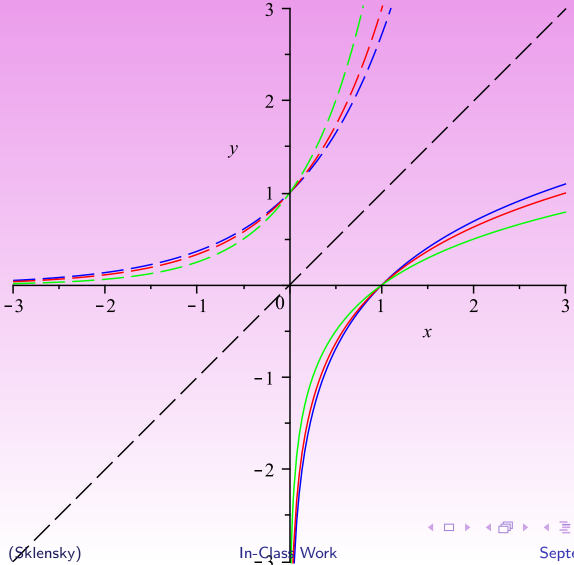

Notice:

- $f(x) = \log_b(x)$ *always* passes through the points $(b, 1)$ and $(1, 0)$, since the points $(1, b)$ and $(0, 1)$ are on the graph of b^x.

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- $f(x) = \log_b(x)$ *always* passes through the points $(b, 1)$ and $(1, 0)$, since the points $(1, b)$ and $(0, 1)$ are on the graph of b^x.
- Logarithms are not defined at $x = 0$:

$$\square = \log_b(0) \Leftrightarrow b^{\square} = 0$$

Thus 0 is not in the domain of any logarithmic function

Also notice:

- Logarithms are not defined at negative numbers:

$$\square = \log_b(\text{negative number}) \Leftrightarrow b^{\square} = \text{ negative number}$$

Since b is positive, no matter what power you raise it to, $b^{\square}$ will always be positive.

Recall:

- Most important log= logarithm base e and is denoted

$$\ln(x) = \log_e(x).$$

- Called the *natural logarithm*
- Later in the semester, we'll see why this is so important.

Algebra of Expontials and Logs

$$b^{-k} = \frac{1}{b^k} \qquad b^{\frac{1}{k}} = \sqrt[k]{b} \qquad b^0 = 1$$

$$b^x b^y = b^{x+y} \qquad (b^x)^y = b^{xy}$$

$$\log_b(xy) = \log_b(x) + \log_b(y) \qquad \log_b(x^k) = k\log_b(x) \qquad \log_b(1) = 0$$

$$\log_b(b^x) = x \qquad b^{\log_b(x)} = x$$

In Class Work

1. Solve $3^{4x-7} = 10$ for x.
2. Solve $3^{4x-7} = 10^{2x+5}$ for x
3. Find a function of the form ae^{bx} which goes through the points $(0, 2)$ and $(2, 6)$.

Solutions to In Class Work

1. Solve $3^{4x-7} = 10$ for x.

$$\begin{aligned}
3^{4x-7} &= 10 \\
\ln\left(3^{4x-7}\right) &= \ln 10 \\
(4x-7)\ln 3 &= \ln 10 \\
4x-7 &= \frac{\ln 10}{\ln 3} \\
4x &= 7 + \frac{\ln 10}{\ln 3} \\
x &= \frac{1}{4}\left(7 + \frac{\ln 10}{\ln 3}\right)
\end{aligned}$$

Solutions to In Class Work

2. Solve $3^{4x-7} = 10^{2x+5}$ for x

$$\begin{aligned} 3^{4x-7} &= 10^{2x+5} \\ (4x-7)\ln 3 &= (2x+5)\ln 10 \\ 4(\ln 3)x - 7\ln 3 &= 2(\ln 10)x + 5\ln 10 \\ \Big(4\ln 3 - 2\ln 10\Big)x &= 5\ln 10 + 7\ln 3 \\ x &= \frac{5\ln 10 + 7\ln 3}{4\ln 3 - 2\ln 10} \end{aligned}$$

Solutions to In Class Work

3. Find a function of the form ae^{bx} which goes through the points $(0, 2)$ and $(2, 6)$.

   $(0, 2)$ on the graph $\Longrightarrow f(0) = 2$.

   That is, $2 = f(0) = ae^{b \cdot 0} = ae^0 = a$.

   $a = 2 \Rightarrow f(x) = 2e^{bx}$.

   We still need to find b.

   $(2, 6)$ on the graph $\Longrightarrow f(2) = 6$.

   That is, $6 = f(2) = 2e^{2b}$.

   We need to solve for b.

$$
\begin{aligned}
6 &= 2e^{2b} \Longrightarrow 3 = e^{2b} \\
\ln(3) &= 2b \text{ (because } \ln(x) \text{ and } e^x \text{ are inverses, so } \ln(e^{2b}) = 2b) \\
\frac{\ln(3)}{2} &= b \Longrightarrow f(x) = 2e^{\frac{x}{2}\ln(3)}
\end{aligned}
$$