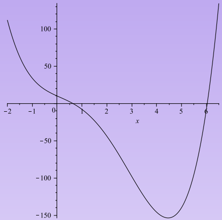
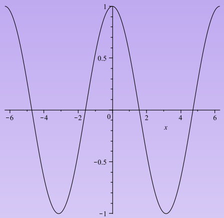
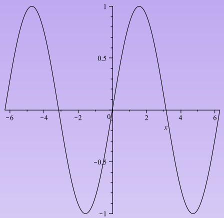
Basic Building Block Functions:



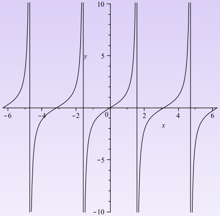
polynomial



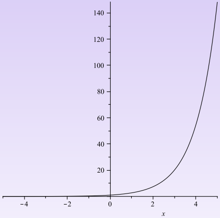
$\cos(x)$



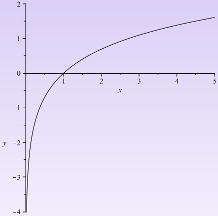
$\sin(x)$



$\tan(x)$



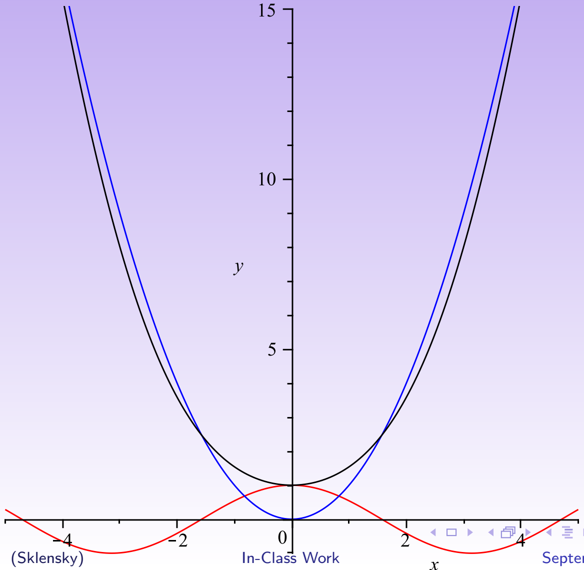
e^x



$\ln(x)$

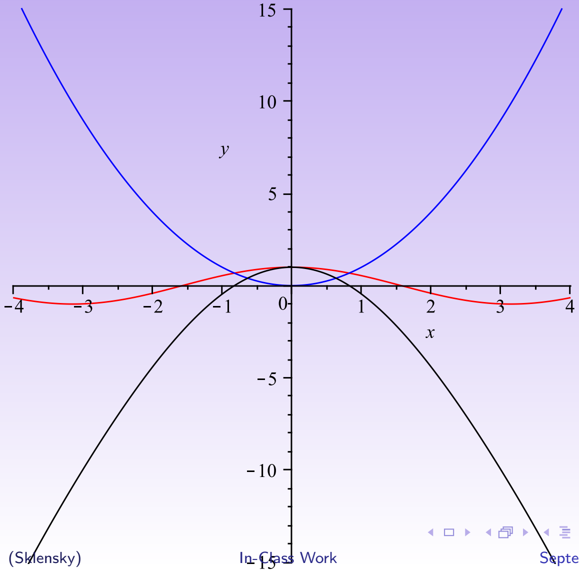
Addition of Functions

$\cos(x)$, x^2, $\cos(x) + x^2$



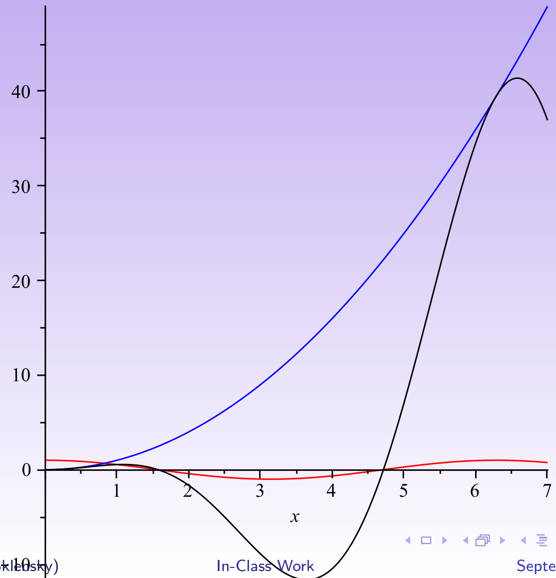
Subtraction of Functions

$\cos(x)$, x^2, $\cos(x) - x^2$



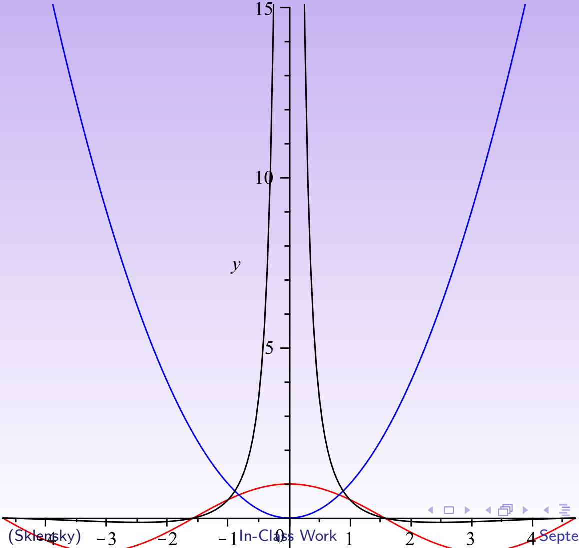
Multiplication of Functions

$\cos(x)$, x^2, $x^2\cos(x)$



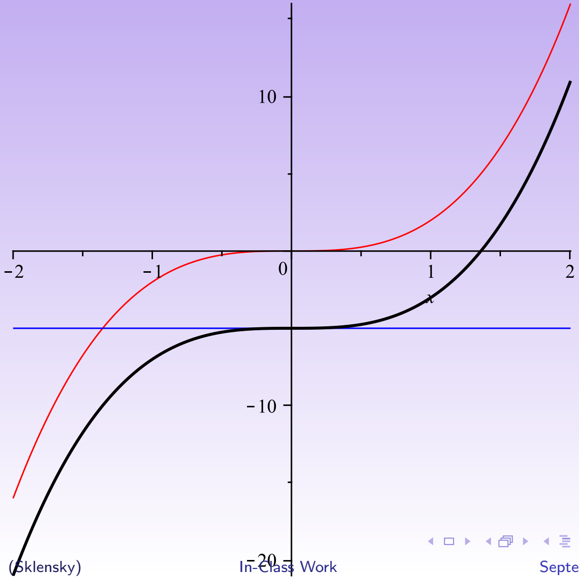
Division of Functions

$\cos(x)$, x^2, $\dfrac{\cos(x)}{x^2}$



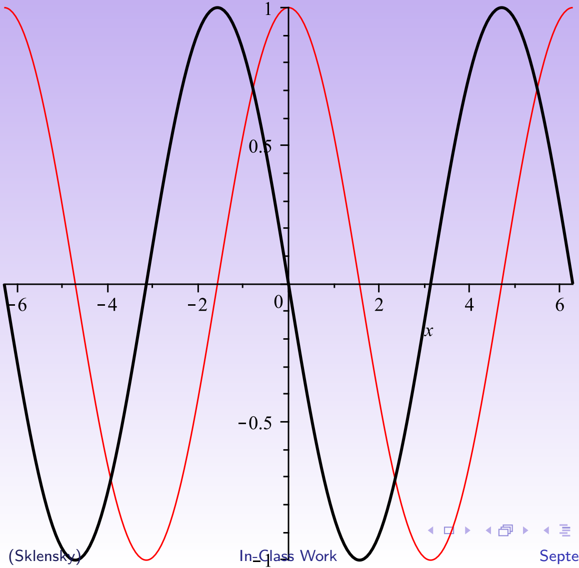
Vertical Translation

$2x^3$, -5, $2x^3 - 5$



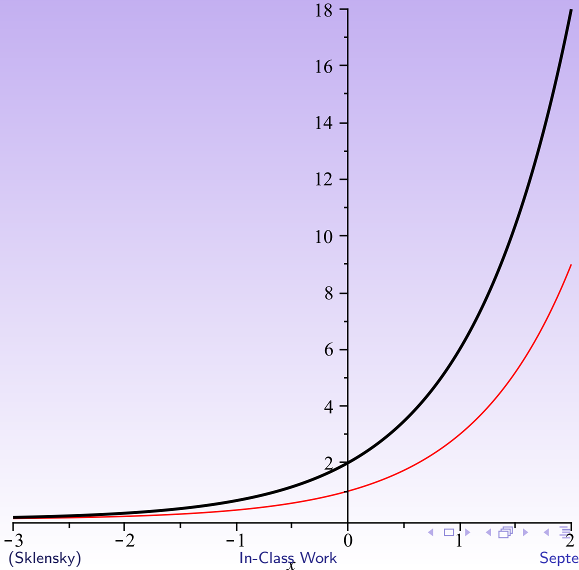
Horizontal Translation

$\cos(x)$, $\cos(x + \pi/2)$



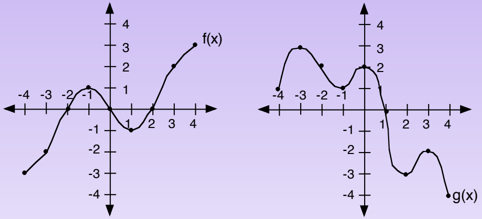
Vertical Stretching

3^x, $2 \cdot 3^x$ (which is not the same as 6^x)



In Class Work

1. Given $f(x) = 2^x$, $g(x) = \cos(x)$, and $h(x) = x^3 - 2x$, find:
   (a) $f \circ g(x)$ (c) $h \circ g(x)$
   (b) $f \circ h(x)$ (d) $f \circ g \circ h(x)$
2. The functions below define $f(x)$ and $g(x)$. Find $f \circ g(-3)$.



3. Let $f(x) = x^2$. In each case, sketch all listed functions on the same set of axes.
   (a) $f(x)$, $f(x) + 2$, and $f(x) - 3$
   (b) $f(x)$, $f(x + 2)$, and $f(x - 3)$
   (c) $f(x)$, $2f(x)$, $-3f(x)$, and $\frac{1}{2}f(x)$

Solutions to In Class Work

1. Given $f(x) = 2^x$, $g(x) = \cos(x)$, and $h(x) = x^3 - 2x$, find:

   (a) $f \circ g(x)$

   $$f \circ g(x) = f\big(g(x)\big) = f\big(\cos(x)\big) = 2^{\cos(x)}.$$

   (b) $f \circ h(x)$

   $$f \circ h(x) = f\big(h(x)\big) = f\big(x^3 - 2x\big) = 2^{x^3-2x}.$$

   (c) $h \circ g(x)$

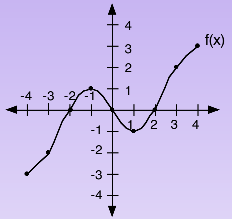
   $$h \circ g(x) = h\big(g(x)\big) = h\big(\cos(x)\big) = \big(\cos(x)\big)^3 - 2\cos(x).$$

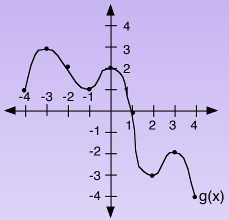
   (d) $f \circ g \circ h(x)$

   $$\begin{aligned} f \circ g \circ h(x) &= f\Big(g\big(h(x)\big)\Big) = f\Big(g\big(x^3 - 2x\big)\Big) = f\Big(\cos(x^3 - 2x)\Big) \\ &= 2^{\cos(x^3-2x)}. \end{aligned}$$

In Class Work

2. The functions below define $f(x)$ and $g(x)$. Find $f \circ g(-3)$.



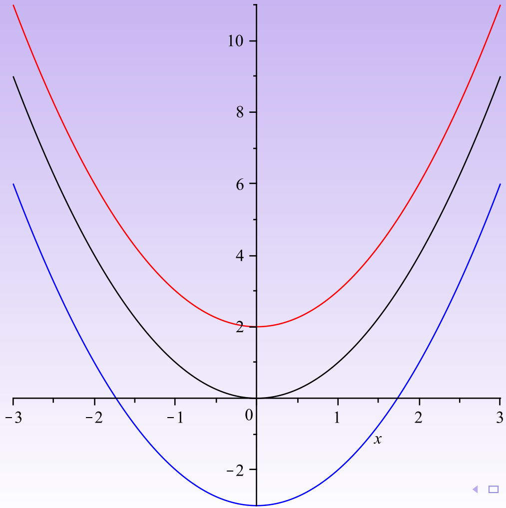


- $f \circ g(-3) = f\big(g(-3)\big)$.
- From the graph of $g(x)$, $g(-3) = 3$.
- Thus $f \circ g(-3) = f\big(g(-3)\big) = f(3)$
- From the graph of $f(x)$, $f(3) = 2$.
- Thus $f \circ g(-3) = 2$.

In Class Work

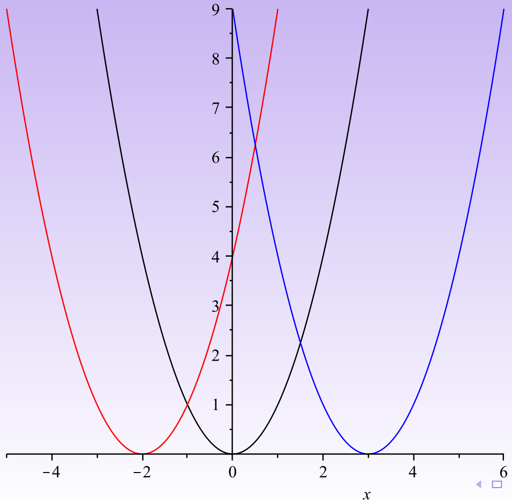
3. Let $f(x) = x^2$. In each case, sketch all listed functions on the same set of axes.

   (a) $f(x)$, $f(x) + 2$, and $f(x) - 3$



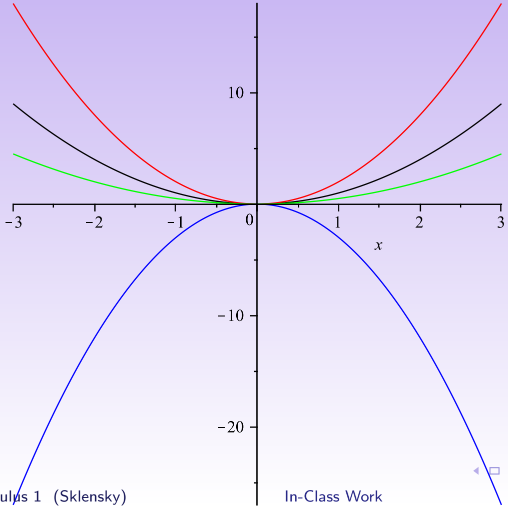
In Class Work

3. Let $f(x) = x^2$. In each case, sketch all listed functions on the same set of axes.
   (b) $f(x)$, $f(x+2)$, and $f(x-3)$



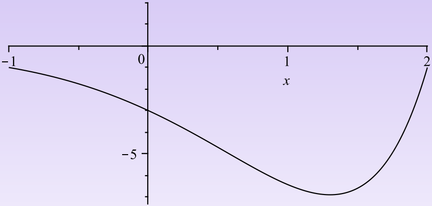
In Class Work

3. Let $f(x) = x^2$. In each case, sketch all listed functions on the same set of axes.
   (c) $f(x)$, $2f(x)$, $-3f(x)$, and $\frac{1}{2}f(x)$



In Class Practice

1. Find the compositions $f \circ g(x)$ and $g \circ f(x)$ if $f(x) = x^2 + x$ and $g(x) = \sin(x)$.
2. Identify functions $f(x)$ and $g(x)$ such that $\sqrt{(x-2)^4 + 3}$ is $f \circ g(x)$.
3. Use the graph below to sketch the graph of $3f(x) + 5$



Solutions:

3. Find the compositions $f \circ g(x)$ and $g \circ f(x)$ if $f(x) = x^2 + x$ and $g(x) = \sin(x)$.
   (a) $f \circ g(x) = f\big(g(x)\big) = f\big(\sin(x)\big) = \sin^2(x) + \sin(x)$
   (b) $g \circ f(x) = g\big(f(x)\big) = g\big(x^2 + x\big) = \sin(x^2 + x)$
4. Identify functions $f(x)$ and $g(x)$ such that $\sqrt{(x-2)^4 + 3}$ is $f \circ g(x)$.
   - One possibility: $g(x) = x - 2$, $f(x) = \sqrt{x^4 + 3}$
   - Another possibiity: $g(x) = (x-2)^4$, $f(x) = \sqrt{x + 3}$
   - Yet another possibility: $g(x) = (x-2)^4 + 3$, $f(x) = \sqrt{x}$.

Solutions:

5. Use the graph below to sketch the graph of $3f(x) + 5$