1. Evaluate the following integrals. In some cases, a sketch of the region may be useful.

   (a) $\int_0^{\pi/2} \cos(x)\, dx$
   $\sin(x)$ is an antiderivative of $\cos(x)$, so from the FTC v2, we know

   $$\int_0^{\pi/2} \cos(x)\, dx = \sin(x) \text{ from } 0 \text{ to } \pi/2 \ = \sin(\pi/2) - \sin(0) = 1.$$

   (b) $\int_1^4 x^3 - 2x\, dx$
   $\frac{x^4}{4} - x^2$ is an antiderivative of $x^3 - 2x$, so from the FTC v2, we know

   $$\int_1^4 x^3 - 2x\, dx = (\frac{x^4}{4} - x^2) \text{ from } 1 \text{ to } 4 = (\frac{4^4}{4} - 16) - (\frac{1}{4} - 1) = 48 + \frac{3}{4}$$

   (c) $\int_{-1}^2 e^x\, dx$
   e^x is of course an antiderivative of e^x, so from the FTC v2, we know

   $$\int_{-1}^2 e^x\, dx = e^x \text{ from } -1 \text{ to } 2 \ = e^2 - e^{-1}$$

   (d) $\int_1^3 3x^2 \ln(x) + x^3 \left(\frac{1}{x}\right)\, dx$
   If $f(x) = 3x^2 \ln(x) + x^3 \left(\frac{1}{x}\right)$, then I know from the FTC that

   $$\int_1^3 3x^2 \ln(x) + x^3 \left(\frac{1}{x}\right) \ ; dx = F(3) - F(1),$$

   where $F(x)$ is any antiderivative of $3x^2 \ln(x) + x^3 \left(\frac{1}{x}\right)$.
   So ...

   - **Goal 1:** find an antiderivative $F(x)$ of $3x^2 \ln(x) + x^3 \left(\frac{1}{x}\right)$.
     - Whatever F might be, it has to differentiate into this sum of two products.

- The product on the right $x^3 \left(\dfrac{1}{x}\right)$ can be simplified to x^2, which I know how to antidifferentiate. I am not going to do it yet, however, in case for some reason it turns out this product and the product on the left are related.
- The two ways we're most familiar with that a function can differentiate into a product is if the original function – the one we're trying to find, the one that antidifferentiates to what we have – is a composition or a product.
- When you differentiate a composition, the result is also a composition.
- There is no composition in $3x^2 \ln(x)$. Therefore this product did not come from differentiating a composition.
- F is most likely a product. In that case, when we differentiate F, we would get a sum of two products. Do we have that inside our integral? Yes!
- The product rule says that if $F = fg$, then $F' = fg' + f'g$ – in other words, the two products that make up the sum are very closely related – each one has an antidifferentiated function (either f or g) and a differentiated function (either g' or f').
- When I look more closely at $3x^2 \ln(x) + x^3 \left(\frac{1}{x}\right)$, I see that $3x^2$ is the derivative of x^3 and $\dfrac{1}{x}$ is the derivative of $\ln(x)$. That tells me that f and g are x^3 and $\ln(x)$.
- Try: $F(x) = x^3 \ln(x)$.
- *Check:* $F'(x) = (x^3)' \cdot \ln(x) + x^3 \cdot (\ln(x))' = 3x^2 \ln(x) + x^3 \ln(x)$
- **Conclusion:** $F(x) = x^3 \ln(x)$.

• **Goal 2:** find the value of the definite integral
Using FTC v2,

$$\int_1^3 3x^2 \ln(x) + x^3 \left(\frac{1}{x}\right)\, dx = F(3) - F(1) = [3(3)^2 \ln(3)] - [3(1)^2 \ln(1)] = 27 \ln(3).$$