Consider the sequence $\left\{\frac{5k^2-42}{3k^2+5}\right\}_{k=1}^{\infty}$. We want to know whether or not this sequence converges, and if so, what to. Just to try to get a feel for what's going on with this sequence, let's look at the first several terms of this sequence.

k	$a_k = \frac{5k^2 - 42}{3k^2 + 5}$
1	$\frac{5 \cdot 1^2 - 42}{3 \cdot 1^2 + 5} = -\frac{37}{8}$
2	$\frac{5 \cdot 2^2 - 42}{3 \cdot 2^2 + 5} = -\frac{22}{17}$
3	$\frac{5\cdot3^2-42}{3\cdot3^2+5} = \frac{3}{32}$
4	$\frac{38}{53}$
5	$\frac{83}{80}$
6	$\frac{138}{113}$

Thus the sequence begins like

$$\left\{-\frac{37}{8}, -\frac{22}{17}, \frac{3}{32}, \frac{38}{53}, \frac{83}{80}, \frac{138}{113}, \ldots\right\}$$

Do the following sequences converge or diverge? If the sequence converges, find the limit.

1.
$$\left\{\frac{j^2 + 32j}{e^j}\right\}_{j=3}^{\infty}$$

2.
$$\left\{\frac{\sin(k)}{k^2}\right\}_{k=1}^{\infty}$$

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 $\mathbf{Sklensky}$

Example: $\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$ is a geometric series, with $r = \frac{1}{2}$.

The associated sequence of terms $\{a_k\}$ is

$$\left\{ \left(\frac{1}{2}\right)^k \right\}_{k=0}^{\infty} = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \}$$

The associated sequence of partial sums S_n is

$$\{1, 1 + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{4}, 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}, \\1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}, \ldots\} = \{1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \ldots\}$$