Recall:

1. **Definition:** An **alternating series** is one whose terms alternate in sign. That is, a series of the form $c_1 - c_2 + c_3 - \cdots$ where c_i is positive.

2. **Alternating Series Test:** Consider the alternating series $c_1 - c_2 + c_3 - \cdots = \sum_{k=1}^{\infty}(-1)^{k+1}c_k$ where $c_k \geq 0$.

   If $\lim_{k\to\infty} c_k = 0$, then the series converges.

   If in addition $c_1 > c_2 > c_3 > \ldots$, then S lies between S_n and S_{n+1} for any n.

Determine whether the following series converge conditionally, converge absolutely, or diverge. For those that converge, find upper and lower bounds.

1. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{k^2+1}$

2. $\displaystyle\sum_{k=2}^{\infty}\frac{(-1)^k}{k\ln(k)}$

3. $\displaystyle\sum_{j=1}^{\infty}\frac{(-1)^j}{(j+1)\sqrt{2j}}$

4. $\displaystyle\sum_{n=1}^{\infty}(-1)^n n\left(\frac{2}{3}\right)^n$

5. $\displaystyle\sum_{k=1}^{\infty}\frac{\cos(k)}{k^4+1}$

Show that the following series converge, and approximate each series accurate to within 0.001.

1. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{k^2+1}$

2. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^5}{n^6+17}$

3. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{4n}{n!+n+2}$

4. $\displaystyle\sum_{j=3}^{\infty}\frac{(-1)^j}{4^j}$