Example: $\displaystyle\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k$ is a geometric series, with $r=\frac{1}{2}$.

The associated **sequence of terms** $\{a_k\}$ is

$$\left\{\left(\frac{1}{2}\right)^k\right\}_{k=0}^{\infty}=\{1,\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16},\ldots\}$$

The associated **sequence of partial sums** S_n is

$$\begin{aligned}\{1,1+\frac{1}{2},1+\frac{1}{2}+\frac{1}{4},1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8},&\\ 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16},\ldots\} &= \{1,\frac{3}{2},\frac{7}{4},\frac{15}{8},\frac{31}{16},\ldots\}\end{aligned}$$

1. For each series below:

   (i) Find a_2 and a_3; S_2 and S_3.

   (ii) Does the series converge or diverge? If it converges, find the value to which it converges.

   (a) $\displaystyle\sum_{k=0}^{\infty} \frac{4}{3^k}$ (b) $\displaystyle\sum_{k=0}^{\infty} \frac{3^k}{(-4)^k}$

2. For each series below, does the series converge or diverge? If it does converge, find the value to which it converges. *Note in each case where the series starts!*

   (a) $\displaystyle\sum_{k=2}^{\infty} \frac{5^k}{2^k}$ (b) $\displaystyle\sum_{k=42}^{\infty} \frac{1}{5^k}$

Do the following series converge or diverge?

1. $\displaystyle\sum_{k=1}^{\infty} \frac{2k^2 - 3}{5k^2 + 6k}$

2. $\displaystyle\sum_{k=98}^{\infty} \frac{3^k + \sin(k)}{\cos(k) + 5}$

3. $\displaystyle\sum_{k=2}^{\infty} \frac{5^k - 6k - 27}{7^k + 14k^2 + k}$