1. The nth Taylor polynomial for $f(x)$ based at $x = x_0$ is

$$\begin{aligned} P_n(x) &= f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 \\ &\quad + \frac{f'''(x_0)}{3!}(x - x_0)^3 \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n \end{aligned}$$

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2. The idea behind Taylor polynomials approximating a function $f(x)$ is to focus on how f behaves at *one point* x_0. We match not only the y-values at x_0, but also the slopes (the first derivative), the concavity (the second derivative), and however many more derivatives we choose – n is the number of derivatives we're choosing to match.

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3. Based on just one example, it seems as if perhaps the higher n is, the better an approximation $P_n(x)$ gives.

Friday, we found that we can approximate $\cos(x)$ near $x = 0$ by

$$\cos(x) \approx 1 - x^2/2! + x^4/4! - x^6/6!$$

and near $x = 2\pi$ by

$$\cos(x) \approx 1 - (x - 2\pi)^2/2! + (x - 2\pi)^4/4! - (x - 2\pi)^6/6!$$

We also saw that the first six derivatives of $\cos(x)$ at $x = 0$ agree with the first six derivatives of
$P_6(x) = 1 - x^2/2! + x^4/4! - x^6/6!$ at $x = 0$.

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2. Rewriting this slightly, the nth Taylor polynomial for $f(x)$ based at $x = x_0$ is

$$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + a_3(x - x_0)^3 \cdots + a_n(x - x_0)^n,$$

where $a_i = \dfrac{f^{(i)}(x_0)}{i!}$.

1. When the base point is $x_0 = 0$, this becomes

$$\begin{aligned} P_n(x) &= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 \\ &\quad + \frac{f'''(0)}{3!}x^3 \cdots + \frac{f^{(n)}(0)}{n!}x^n \end{aligned}$$

   A Taylor polynomial based at $x = 0$ is also called a *MacLaurin* polynomial.

2. Again, rewriting this,

$$\begin{aligned} P_n(x) &= a_0 + a_1 x + a_2 x^2 \\ &\quad + a_3 x^3 \cdots + a_n x^n, \end{aligned}$$

   where $a_i = \dfrac{f^{(i)}(0)}{i!}$.

1. (a) Find the 3rd, 5th, and 7th Taylor polynomial for $f(x) = \sin(x)$ based at $x_0 = 0$

   (b) Check how good an approximation $P_3(x)$, $P_5(x)$, and $P_7(x)$ are by graphing $P_3(x)$, $P_5(x)$, $P_7(x)$ and $\sin(x)$ all on the same set of axes. (Find an interval that gives you a sense of where the approximations are good and where they are not.)

   (c) Approximate $\sin\left(\frac{1}{2}\right)$ using $P_7(x)$. Compare it to the approximation Maple gives for $\sin\left(\frac{1}{2}\right)$.

2. Find the 6th Taylor polynomial for $g(x) = e^x$ based at $x = 0$, and use it to approximate e.