1. For each integral, explain why $\int_a^b f(x)\,dx$ is improper, and determine whether the integral converges or diverges.

   a. $\int_1^\infty \frac{1}{x^3}\,dx$

   b. $\int_1^\infty \frac{1}{x}\,dx$

   c. $\int_1^\infty 1+\frac{1}{x^2}\,dx$

   d. $\int_1^\infty \frac{1}{x^p}\,dx$ where $p > 1$

2. Think about the above results and the big picture of what's going on.

   (a) Is it *necessary* that $f(x)$ converge to 0 as $x \to \infty$ in order for $\int_a^\infty f(x)\,dx$ to converge to a finite number?

   (b) If $f(x)$ does converge to 0 as $x \to \infty$, *must* $\int_a^b f(x)\,dx$ automatically converge to a finite number? That is, is $f(x) \to 0$ a *sufficient* condition for $\int_a^\infty f(x)\,dx$ to converge to a finite number?