Recall:

- **Definition:** We define an *improper integral* of the form $\int_a^\infty f(x)\,dx$ as follows:

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx.$$

  The integral on the right is a proper definite integral over a finite interval, and so the Fundamental Theorem of Calculus applies.

- **Definition:** We say an improper integral $\displaystyle\int_a^\infty f(x)\,dx$ *converges* if

$$\lim_{b\to\infty} \int_a^b f(x)\,dx$$

  exists and converges to a finite number. Otherwise, we say that the improper integral *diverges*.

- $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ converges if $p > 1$, diverges if $p \le 1$.

  The same results are true for $\displaystyle\int_a^\infty \frac{1}{x^p}\,dx$ for any $a > 0$.

Leftover from Thursday:

2. Think about the results you saw yesterday, and the big picture of what's going on.

   (a) Is it *necessary* that $f(x)$ converge to 0 as $x \to \infty$ in order for $\int_a^\infty f(x)\,dx$ to converge to a finite number?

   (b) If $f(x)$ does converge to 0 as $x \to \infty$, *must* $\int_a^b f(x)\,dx$ automatically converge to a finite number? That is, is $f(x) \to 0$ a *sufficient* condition for $\int_a^\infty f(x)\,dx$ to converge to a finite number?

Determine whether the integral converges or diverges.

1. $\int_0^1 \frac{1}{\sqrt{x}}\, dx$

2. $\int_0^1 \frac{1}{x}\, dx$

3. $\int_0^1 \frac{1}{x^3}\, dx$

4. $\int_0^1 \frac{1}{x^p}\, dx$

Determine whether the integral converges or diverges.

1. $\displaystyle\int_{0}^{\infty} xe^{-x}\, dx$
2. $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}\, dx$
3. $\displaystyle\int_{-2} 2\frac{1}{x^2}\, dx$
4. $\displaystyle\int_{1}^{\infty} \frac{1}{x^2+5}\, dx$