## Determining Convergence

**Key Step:** Identify where in the interval the integral is improper, and convert to a limit as we approach that number.

**Examples:**

1. $$\int_3^\infty \frac{1}{(x-1)^4}\,dx = \lim_{b\to\infty}\int_3^b \frac{1}{(x-1)^4}\,dx$$

2. $$\int_2^{100} \frac{x}{\sqrt{x^2-4}}\,dx = \lim_{a\to 2}\int_a^{100} \frac{x}{\sqrt{x^2-4}}\,dx$$

**Determining Convergence - Important Reminders**

Consider $I = \int_a^\infty f(x)\,dx$.

1. There is a huge distinction between $f(x)$ converging – that is, $\lim_{x\to\infty} f(x)$ being finite – and $I$ converging. Just because you can find $\lim_{x\to\infty} f(x)$, and it's a finite number, does **not** mean that $\int_a^\infty f(x)\,dx$ will be finite.
2. In fact, if $\lim_{x\to\infty} f(x)$ exists but is not 0, $I$ diverges! No need to investigate any further.
3. If $\lim_{x\to\infty} f(x) = 0$, $I$ may converge or it may diverge – you must investigate further.

## Determining Convergence

**Question:** How do we determine whether or not an improper integral converges if we can't find the antiderivative?

And if it *does* converge, how would we approximate the value of the improper integral?

Determine whether each of the following improper integrals converges or diverges.

1. $\displaystyle\int_2^\infty \frac{1}{x^3+2}\,dx$

2. $\displaystyle\int_5^\infty \frac{1}{\sqrt{x}-2}\,dx$

3. $\displaystyle\int_2^\infty \frac{2}{\sqrt{x}+x^2}\,dx$

4. $\displaystyle\int_0^\infty \frac{2}{\sqrt{x}+x^2}\,dx$

## Goals:

1. Is there any way to at least determine whether or not an improper integral $I$ converges even if we cannot find an antiderivative?
2. Better yet, if we *do* determine that an improper integral $I$ converges, is there a way to approximate the value of the integral $I$?