Goal: Figure out what to do when faced with an improper integral that we can't evaluate simply by taking limits and antidifferentiating.

1. Determine first whether or not the improper integral converges, by comparing it in a useful way to some other integral whose convergence or divergence we know.

2. If the improper integral *does* converge, approximate it.

Let $I = \int_a^\infty f(x)\,dx$.

Dealing with Goal 1:

1. If $f(x)$ **is** antidifferentiable, cope with I by taking the limit of proper definite integrals. This tells us whether I diverges or converges, and if so, what it converges to.

2. If $f(x)$ is **not** antidifferentiable, then we try to determine whether or not I converges by comparing it to an improper integral whose convergence or divergence we know:

   (a) If I is *less* than or equal to a convergent improper integral (but greater than or equal to 0), it must converge also. If it is *greater than* a convergent improper integral, our comparison was useless.

   (b) If I is *greater* than or equal to a (positive) divergent improper integral, then it must diverge also. If it is *less than* a divergent improper integral, our comparison was useless.

Still left to figure out: Goal 2

If the integrand of an improper integral is *not* antidifferentiable, and you've already determined the improper integral converges, how can you approximate what it converges to?

Plan for approximating a convergent improper integral:

1. **Replace the improper integral with a proper one**

   Replace $\int_a^\infty f(x)\, dx$ with $\int_a^t f(x)\, dx$.

   The error will be the *tail*, $\int_t^\infty f(x)\, dx$.

   Because the bigger t is, the smaller the tail is, we can control the error introduced by this replacement by making t sufficiently large.

2. **Approximate the proper integral with one of our usual techniques**

   Approximate $\int_a^t f(x)\, dx$ using left, right, midpoint, or trapezoidal sums. This will of course introduce another error.

In each case, determine first whether the improper integral converges or diverges. If it converges, find a definite integral which is within .01 of the improper integral. If it diverges, find a t so that $\displaystyle\int_a^t f(x)\,dx > 100$.

1. $\displaystyle\int_1^{\infty} \frac{5}{x^3 + e^x}\,dx$
2. $\displaystyle\int_4^{\infty} \frac{1}{x^{1/2} - x^{1/3}}\,dx$