1. Differentiate the following functions.

   (a) $f(x) = \ln(x^2 + 3)$
   chain rule!

$$f'(x) = \frac{1}{x^2+3} \cdot 2x$$

   (b) $g(w) = w\cos(e^w)$
   product rule and chain rule!

$$g'(w) = w \cdot -\sin(e^w) \cdot e^w + \cos(e^w)$$

   (c) $h(s) = \dfrac{s^{-3} - \pi}{\sqrt{s}}$
   quotient rule!

$$h'(s) = \frac{\sqrt{s} \cdot (-3s^{-4} + 0) - (s^{-3} - \pi) \cdot \frac{1}{2}s^{-1/2}}{(\sqrt{s})^2} = \frac{\dfrac{-3\sqrt{s}}{s^4} - \dfrac{s^{-3} - \pi}{2\sqrt{s}}}{s}$$

2. Find an antiderivative for each of the following functions.

   (a) $p(x) = 3x^5 + 7x^4 - \dfrac{x^2}{3} + 11$
   An antiderivative is

$$P(x) = \frac{3}{6}x^6 + \frac{7}{5}x^5 - \frac{1}{3} \cdot \frac{1}{3}x^3 + 11x$$

   Notice: I didn't put in a

   `+C`

   because I simply asked to find *an* antiderivative. The C implies that you're talking about the family of *all* antiderivatives.

   (c) $v(t) = 2e^t - 3\cos(3t)$
   An antiderivative is

$$V(t) = 2e^t - \sin(3t)$$

I can check this by differentiating, and remembering that I have to use the chain rule. If you've already learned (and remember) substitution, you can use that method. Also, if you remember about horizontal and vertical stretching, you can use that. If not, never fear –just use the always popular guess and check method.

3. Suppose that $f(x) = x^2 - 3e^x + 4$. Let $F(x)$ be an antiderivative of $f(x)$.

(a) Is the graph of $f(x)$ increasing or decreasing at $x = -2$?

$$f'(x) = 2x - 3e^x \qquad f'(-2) = -4 - 3e^{-2} < 0 \Rightarrow f \text{ decreasing at } x = -2$$

(b) Is the graph of $f(x)$ concave up or concave down at $x = -2$?

$$f''(x) = 2 - 3e^x \qquad f''(-2) = 2 - 3e^{-2} > 0 \Rightarrow f \text{ concave up at } x = -2$$

(c) Is the graph of $F(x)$ increasing or decreasing at $x = -2$?
I need to see whether $F'(x)$ is positive or negative. But since F is an antiderivative of f, the derivative of F is f.

$$F'(x) = f(x) = x^2 - 3e^x + 4 \qquad F'(-2) = 4 - e^{-2} + 4 > 0 \Rightarrow F \text{ increasing at } x = -2$$

(d) Is the graph of $F(x)$ concave up or concave down at $x = -2$?
I need to see whether $F''(x)$ is positive or negative. But $F''(x) = f'(x)$.

$$F''(-2) = f'(-2) = -4 - 3e^{-2} < 0 \Rightarrow F \text{ concave up at } x = -2$$

4. Find the signed areas given by the following integrals:

(a) $\displaystyle\int_1^4 \pi - x^{-3/2}\, dx$
FTC, v2 says that if we find an antiderivative F, then this signed area is $F(4) - F(1)$.
Thus,

$$\int_1^4 \pi - x^{-3/2}\, dx = \left[\pi x - \frac{x^{-1/2}}{-1/2}\right]_1^4 = \left[\pi x + \frac{2}{\sqrt{x}}\right]_1^4 = (4\pi + 1) - (\pi + 2) = 3\pi - 1$$

(b) $\displaystyle\int_2^3 6z^5 + \frac{5}{z^{10}}\,dz$

$$\int_2^3 6z^5 + \frac{5}{z^{10}}\,dz = \left[z^6 - \frac{5}{9}z^{-9}\right]_2^3 = \left(3^6 - \frac{5}{9\cdot 3^9}\right) - \left(2^6 - \frac{5}{9\cdot 2^9}\right)$$