Let $I = \int_0^1 x\sin(x^2)\ dx$

1. Write out L_5 without Sigma Notation.

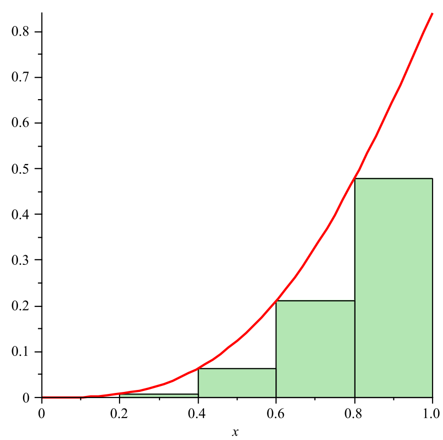
   To partition the interval $[0,1]$ into 5 equal subintervals, we need each subinterval to have width

$$\Delta x = \frac{1-0}{5} = .2.$$

   Thus our partition is $0 < .2 < .4 < .6 < .8 < 1.0$.

   Since we're looking at a *left* sum, the heights of the rectangles will be determined by teh left endpoints of each subinterval – that is, by the heights at $x = 0$, $x = .2$, $x = .4$, $x = .6$, and $x = .8$.

   Thus our left sum with 5 subintervals will look like:



   L_5 is the sum of the areas of these 5 rectangles. The base of each rectangle is .2, while the height of each is f(left endpoint).

$$\begin{aligned} L_5 &= .2\cdot f(0) + .2\cdot f(.2) + .2\cdot f(.4) + .2\cdot f(.6) + .2\cdot f(.8) \\ &= .2\cdot 0 + .2\cdot(.2\sin(.04)) + .2\cdot(.4\sin(.16)) + .2\cdot(.6\sin(.36)) + .2\cdot(.8\sin(.64)) \\ &= .2(0 + .2\sin(.04) + .4\sin(.16) + .6\sin(.36) + .8\sin(.64)) \end{aligned}$$

2. Use Sigma notation to write L_5.

   The patterns in L_5 are possibly easiest to see in the first line of part (a). They will be even easier to see if we work with fractions rather than decimals. In addition to that conversion, I will also factor out that factor of .2 or $\frac{1}{5}$.

$$L_5 = \frac{1}{5}\left(f(0) + f\left(\frac{1}{5}\right) + f\left(\frac{2}{5}\right) + f\left(\frac{3}{5}\right) + f\left(\frac{4}{5}\right)\right).$$

We can see that every term in the sum inside the parentheses is exactly the same exact that the numerators in the fraction are all consecutive numbers from 0 to 4. Thus we're adding up the term

$$f\left(\frac{i}{5}\right)$$

over and over again as i goes from 0 to 4. Thus in sigma notation, we have:

$$L_5 = \frac{1}{5}\sum_{i=0}^{4} f\left(\frac{i}{5}\right).$$

Remember that sigma notation is just shorthand notation for a sum. As I mentioned in class, this can mean that two different people write down the exact same sum using different shorthand. It also means that the rules that apply to sums still apply.

For instance, you could use the distributive property to multiply every term inside the sum represented by the sigma notation to get

$$L_5 = \sum_{i=0}^{4} \frac{1}{5} f\left(\frac{i}{5}\right).$$

3. Calculate the numerical value of L_5. Without finding the exact value of I, decide whether L_5 over-estimates or under-estimates I.

$$L_5 = .2(0 + .2\sin(.04) + .4\sin(.16) + .6\sin(.36) + .8\sin(.64)) = 0.1521692085.$$

Because the height of each rectangle is at or below the height of the function over the same subinterval, the area of each rectangle is less than the area under the function over the subinterval, and so the left sum is an *under-estimate* for I.

4. Write L_{10} and L_{50} using sigma notation.

   You'll be seeing this again on Wednesday.