

Let  $I = \int_0^1 x \sin(x^2) dx$

1. Use the `leftbox` and `rightbox` commands in Maple to look at  $L_{10}$  and  $R_{10}$ .

```
with(student):  
f:= x -> x*sin(x^2);  
leftbox(f(x), x=0..1, 10);  
rightbox(f(x),x=0..1, ,10);
```

2. Write  $L_{10}$  and  $L_{50}$  using sigma notation (without using Maple to help you with the sum).
3. Write  $R_{10}$  and  $R_{50}$  using Sigma notation (again, without using Maple).
4. Without calculating any of them, rank  $I$ ,  $L_{10}$  and  $R_{10}$  in increasing order.
5. Can you draw any conclusions about how well  $L_{10}$  approximates  $I$  (without calculating  $I$ )?

6. Use the formal definition of the integral to write

$$I = \int_0^1 x \sin(x^2) dx \text{ as a limit.}$$