Find the interval of convergence for the following power series:

1. $\displaystyle\sum_{j=0}^{\infty} \frac{x^j}{j!}$
2. $\displaystyle\sum_{n=0}^{\infty} (n+1)(x-3)^n$

Recall:

Theorem: The nth Taylor polynomial for $f(x)$ based at $x = x_0$ is given by $P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + a_3(x - x_0)^3 \cdots + a_n(x - x_0)^n$, where $a_i = \dfrac{f^{(i)}(x_0)}{i!}$

Let $f(x) = \sin(x)$.

1. Find $P_6(x)$ for $f(x)$ based at $x_0 = \frac{\pi}{2}$.

$$\begin{array}{lll} f(x) = \sin(x) & f(\pi/2) = 1 & a_0 = 1/0! = 1 \\ f'(x) = \cos(x) & f'(\pi/2) = 0 & a_1 = 0 \\ f''(x) = -\sin(x) & f''(\pi/2) = -1 & a_2 = -1/2! \\ f'''(x) = -\cos(x) & f'''(\pi/2) = 0 & a_3 = 0/3! = 0 \\ f^{(4)}(x) = \sin(x) & f^{(4)}(\pi/2) = 1 & a_4 = 1/4! \\ f^{(5)}(x) = \cos(x) & f^{(5)}(\pi/2) = 0 & a_5 = 0 \\ f^{(6)}(x) = -\sin(x) & f^{(6)}(\pi/2) = -1 & a_6 = -1/6! \end{array}$$

$$P_6(x) = 1 - \frac{(x - \pi/2)^2}{2!} + \frac{(x - \pi/2)^4}{4!} - \frac{(x - \pi/2)^6}{6!}$$

Find Taylor Series about $x_0 = 0$ for the following:

1. $f(x) = \sin(x)$
2. $g(x) = \cos(x)$
   Hint: $\frac{d}{dx}\sin(x) = \cos(x)$