# Recall:

When $a(x)$ is continuous, non-negative, and decreasing, the value of a convergent series $S$ will be at least as large as any partial sum $S_N$. This means that the actual error incurred by using the $N$th partial sum $S_N$ to approximate the series $S$ is simply $S - S_N$.

Also recall we introduced the sequence of remainders $\{R_N\}$:

$$\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{N} a_k + \sum_{k=N+1}^{\infty} a_k \qquad \text{or} \qquad S = S_N + R_N.$$

Using these concepts,

$$\text{Error using } S_N \text{ to approximate } S = S - S_N = R_N \stackrel{def}{=} \sum_{k=N+1}^{\infty} a_k.$$

Using the same ideas we used to show the Integral Test is true, we found an error bound for the error in using $S_N$ to approximate $S$:

$$\text{Errror using } S_N \text{ to approximate } S = R_N \leq \int_N^\infty a(x)\, dx.$$

# Example: Using the integral test to approximate

Approximate $\displaystyle\sum_{i=4}^{\infty} \frac{2i^3}{(2i^4-14)^2}$ within .0001.

$\displaystyle\sum_{i=4}^{\infty} \frac{2i^3}{(2i^4-14)^2}$ can be shown to converge, using the integral test.

To make $R_N \leq .0001$, set $\displaystyle\int_N^{\infty} \frac{2x^3}{2x^4-14)^2}\,dx \leq .0001$ and solve for $N$.

$$\begin{aligned}
\int_N^{\infty} \frac{2x^3}{(2x^4-14)^2}\,dx \leq .0001 \quad &\Rightarrow \quad \lim_{R\to\infty} -\frac{1}{4}\left(\frac{1}{2x^4-14}\right)\Bigg|_N^R \leq \frac{1}{10000} \\
&\Rightarrow \quad \frac{1}{4(2N^4-14)} \leq \frac{1}{10000} \\
&\Rightarrow \quad 2514 \leq 2N^4 \\
&\Rightarrow \quad N \geq (1257)^{1/4} \approx 5.954 \Rightarrow \text{Use } N=6
\end{aligned}$$

Therefore, $\displaystyle\sum_{i=4}^{\infty} \frac{2i^3}{(2i^4-14)^2} \approx \sum_{i=4}^{6} \frac{2i^3}{(2i^4-14)^2} = (\text{ using Maple }).000745.$

1. You've shown that $S = \sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$ converges.

    1.1 Use the integral test to find lower and upper limits for the value of $S$.

    1.2 Find a number $N$ such that the partial sum $S_N$ approximates the sum of the series within 0.001.

2. Determine whether the series $\sum_{j=2}^{\infty} \frac{1}{j(\ln(j))^5}$ converges or diverges. If the series converges, find a number $N$ such that the partial sum $S_N$ approximates the sum of the series within .001. If the series diverges, find a number $N$ such that $S_N \geq 1000$.

# Recall:

**Goals:** Be able to :

1. determine whether a series $\sum a_k$ converges or diverges.
2. If it converges, find the limit (that is, the value of the series) exactly, if possible.
3. If it converges but we can't find the limit exactly, be able to approximate it.

# Methods we have so far:

Given a series $\sum_{k=M}^{\infty} a_k$,

- **Is it a geometric series?** If so, we can determine whether or not it converges, and if so, exactly what it converges to.

- **kth Term Test:** If $\lim_{k\to\infty} a_k \neq 0$, the series diverges. If $\lim_{k\to\infty} a_k = 0$, inconclusive.

- **Integral Test:** The series does whatever the associated integral $\int_M^{\infty} a(x)\,dx$ does. If the series converges, we can use $R_N \leq \int_N^{\infty} a(x)\,dx$ to approximate the series to any desired degree of accuracy.

Determine the convergence or divergence of the following three series:

1. $$\sum_{k=2}^{\infty} \frac{3^k}{5^k + 2k}$$
2. $$\sum_{k=2}^{\infty} \frac{2k}{7k + 18}$$
3. $$\sum_{j=5}^{\infty} \frac{j!}{(j+2)!}$$