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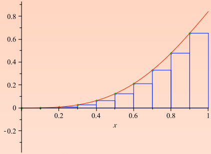
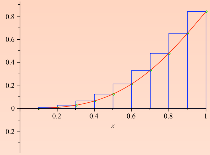
- Using Maple, I can find that
  - $L_{1000} = 1.940005805$
  - $R_{1000} = 1.939463750$
  - $M_{1000} = 1.939734888$
  - And since $T_{1000} = \dfrac{R_{1000} + L_{1000}}{2}$, $T_{1000} = 1.939734778$

  But which of these is the best? How close is each to the actual value of I?

If f is increasing:

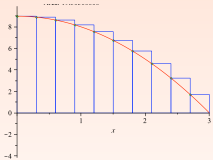


L_n underestimates

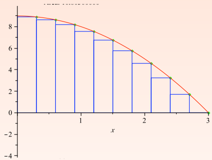


R_n overestimates

If f is decreasing:



L_n overestimates



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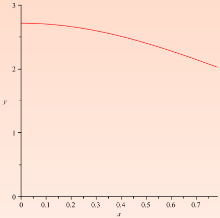

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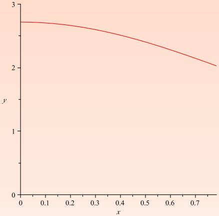

f decreasing on $[0, \pi/4]$

$\Rightarrow |I - L_{1000}| \le |R_{1000} - L_{1000}|$.

- Since $R_{1000} = 1.939463750$

$$\Rightarrow \qquad \text{error} = |I - L_{1000}| \le 0.000542055.$$

- **Example** (continued) $I = \int_0^{\pi/4} e^{\cos(x)}\, dx$.

  Also have no idea how close $M_{1000} = 1.939734888$ is to I.

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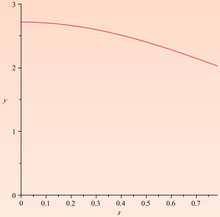
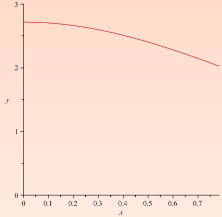
- 

  f concave down on $[0, \pi/4]$
  $\Rightarrow |I - M_{1000}| \leq |T_{1000} - M_{1000}|.$

- **Example** (continued) $I = \int_0^{\pi/4} e^{\cos(x)}\, dx$.

  Also have no idea how close $M_{1000} = 1.939734888$ is to I.

- 

  f concave down on $[0, \pi/4]$
  $\Rightarrow |I - M_{1000}| \leq |T_{1000} - M_{1000}|$.

- Since $T_{1000} = 1.939734778$,

$$\text{error} = |I - M_{1000}| \leq 0.000000011.$$

Definition:

Let $[a, b]$ be partitioned into n equal subintervals by $n + 1$ points

$$a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$$

and let Δx be the width of the each subinterval. In the ith subinterval, pick a point c_i. A **Riemann sum** for f and this partition is

$$f(c_1)\Delta x + f(c_2)\Delta x + \cdots + f(c_n)\Delta x = \sum_{i=1}^{n} f(c_i)\Delta x$$

Formal Definition of the Integral:

$$\int_a^b f(x)\, dx \stackrel{def}{=} \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

Let $I = \int_0^1 x\sin(x^2)\, dx$

1. Use the RiemannSum command in Maple to look at L_{10} and R_{10}.
   - Load the student package: *Tools-Load Package-Student Calculus 1*.
   - Type in:

```
f:= x -> x*sin(x^2);
RiemannSum(f(x), x=0..1, partition=10,method=left,
   output=plot);
RiemannSum(f(x),x=0..1, partition=10, method=right,
   output=plot);
```

2. Write L_{10} and L_{50} using sigma notation (without using Maple).
3. Write R_{10} and R_{50} using Sigma notation (again, without Maple).
4. Use the formal definition of the integral to write $I = \int_0^1 x\sin(x^2)\, dx$ as a limit.