A Guide to Writing in Mathematics Classes

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September 28, 2007

Contents

0.1 **Why Should You Have to Write Papers in a Math Class?** 2

0.2 **How is Mathematical Writing Different from What You've Done So Far?** 2

0.3 **Good Phrases to Use in Math Papers:** 4

0.4 **Phrases to Avoid (or at least think twice about using) in Math Papers:** 4

0.5 **Following the Checklist:** 5

0.1 Why Should You Have to Write Papers in a Math Class?

Up to this point in your mathematical career, most likely the only kind of writing you've done in math classes has been on problem sets and tests, and up to this point you've explained your work to people that know more mathematics than you do (that is, to your teachers). But soon, this will change.

Now that you are taking Calculus, you know far more math than the average American has ever learned and more math than most college graduates remember. With each additional math course you take, you further distance yourself from the average person on the street. You may feel like the math you can do is simple and obvious (doesnt everybody know what a function is?), but you can be sure that other people find it bewilderingly complex. It becomes increasingly important, therefore, that you can explain what you're doing to others that might be interested: your parents, your boss, the media.

Nor are mathematics and writing far removed from one another. Professional mathematicians spend most of their time writing: communicating with colleagues, applying for grants, publishing papers, writing memos and syllabi. Writing well is extremely important to mathematicians, since poor writers have a hard time getting published, getting attention from the Deans, and obtaining funding. It is ironic but true that most mathematicians spend more time writing than they spend doing math.

But most of all, one of the simplest reasons for writing in a math class is that writing helps you to learn mathematics better. By explaining a difficult concept to other people, you end up explaining it to yourself.

> *Every year, we buy ten cases of paper at $35 each; and every year we sell them for about $1 million each. Writing well is very important to us.*
>
> - Bill Browning, President of *Applied Mathematics, Inc.*

0.2 How is Mathematical Writing Different from What You've Done So Far?

A good mathematical essay has a fairly standard format. This section will give you an overview of that format, and how it differs from writing in other subjects.

We tend to begin an essay or letter that addresses solving a problem by first explaining what the problem is. On your problem sets, you've usually just said, 9(a) and then plunged ahead; in your formal writing, you'll have to take much greater pains. Even if the person who is reading your essay is the one who presented you with the problem, you should restate it. This serves several purposes: it reminds the reader of both the background context and the problem at hand, it tells him or her what *you* understood the problem to be, and it helps frame the rest of the essay by serving as an introduction. You should phrase this introduction in a way that will convince your audience that it's an interesting or worthwhile problem to solve.

In much of the writing you've done to date, you've probably been discouraged from repeating yourself. Because mathematical ideas are often complicated, we indulge in some creative repetition in mathematical writing. For instance, we discuss the solution twice – first, we discuss the plan of attack we'll be using in broad terms and everyday language. Later, we follow up with mathematical detail.

It's a good idea to discuss your plan of attack right after stating (or restating) the problem. We want to give the people who are reading our essays an overview – in easily understood everyday language – of what we're going to be discussing in detail later. Rather than simply listing the steps we're going to follow, this often involves stepping back and looking at the big picture: if the problem is one that was originally phrased in a real-life setting, why did we decide to approach it mathematically, and specifically why did we choose to use these specific techniques? Begin with the initial big ideas, and proceed with plan where each idea leads logically to the next – but still all (or mostly) in everyday language.

After the plan of attack but before the mathematical detail, we usually then briefly state the answer, in everyday terms if possible. This gives the audience the confidence to proceed, as the plan clearly leads to a result. (It also gives the lazy reader enough information to quit reading at that point, but we don't assume our readers are lazy.) It's actually fairly uncommon, although not so uncommon as to be exceptional, to read a math paper in which the answer is left for the very end. Explaining the solution *first* and *then* stating the answer is usually reserved for cases where the solution technique is even more interesting than the answer, or when the writers want to leave the readers in suspense. But if the solution is complicated, or simply if you think the reader is eager to find out the answer, then its typically best to hook the readers with the answer before presenting them with the in details. While we hope our presentation of the details will be interesting and exciting, there's no denying that sometimes readers get bogged down in the details.

Only after stating the solution do we proceed to describe in detail how we found the solution.

Another difference is that when you do your homework, it's important to show exactly how you got your answer. However, when you write to a non-mathematician, sometimes it's better to show why your answer works, with just a brief explanation as to how you got it. For example, compare:

Homework Mathematics:

To solve for when $3x^2 - 21x + 30 = 0$, we use the quadratic formula:

$$\begin{aligned} x &= \frac{21 \pm \sqrt{21^2 - 4 \cdot 3 \cdot 30}}{2 \cdot 3} \\ &= \frac{21 \pm \sqrt{441 - 360}}{6} \\ &= \frac{21 \pm 9}{6} \\ &= \frac{30}{6} \text{ or } \frac{9}{6} \\ &= 5 \text{ or } 2, \end{aligned}$$

and so either $x = 5$ or $x = 2$.

More Formal Mathematics:

To solve for when $3x^2 - 21x + 30 = 0$, we used the quadratic formula and found that either $x = 5$ or $x = 2$. It's easy to see that these are in fact the values of x that make $3x^2 - 21x + 30$ zero because

$$3\dot{(}5)^2 - 21 \cdot 5 + 30 = 75 - 105 + 30 = 0,$$

and also

$$3 \cdot (2)^2 - 21 \cdot 2 + 30 = 12 - 42 + 30 = 0.$$

Notice that the formal presentation of the mathematics both gives the interested and qualified reader enough information so that they can reproduce your calculations if necessary *and* contains an explanation of how you check that the two values for x are correct – a process that was not included in the homework presentation. What it does not contain is the specific arithmetic process you followed. The difference is that, in the first example, you're trying to convince someone who knows a lot of math that you, too, know the techniques you're supposed to (and if you make a mistake, there's enough information so that the teacher can determine where the mistake occurred). In the second example, **you're trying to convince someone who may or may not be good at math that you did indeed get the right answer.**

Math is difficult enough that the writing around it should be simple. 'Beautiful' math papers are the ones that are the easiest to read: clear explanations, uncluttered expositions on the page, well-organized presentation. For that reason, mathematical writing is not a creative endeavor the same way that, say, poetry is: you shouldn't be spending a lot of time searching for the perfect word, but rather should be developing the most clear exposition. Unlike humanities students, mathematicians don't have to worry about over-using 'trite' phrases in mathematics. In fact, the next section contains a list of trite but useful phrases that you may want to use in your papers, either in this class or in the future.

This guide, together with your checklist, should serve as a reference while you write. The final section contains explanations of each item on the checklist. If you can master these basic areas, your writing may not be spectacular, but it should be clear and easy to readwhich is the goal of mathematical writing, after all.

0.3 Good Phrases to Use in Math Papers:

- *Therefore* (also: *so, hence, accordingly, thus, it follows that, we see that, from this we get, then*)
- *I am assuming that* (also: *assuming, where, M stands for ...*; in more formal mathematics, we might say *Suppose M is ..., If M represents ..., or Given that,*)
- *show* (also: *demonstrate, prove, explain why, find*
- *This formula can be found on page 433 of* Calculus from Graphical, Numerical, and Symbolic Points of View*, ©2002, Ostebee and Zorn.*
- *If you have any further questions, feel free to contact me.*
- *(see the formula above).* (also: *(see *), this tells us that ...*)
- *if* (also: *whenever, provided that, when*)
- *notice that* (also: *note that, notice, recall*)
- *since* (also: *because*)

0.4 Phrases to Avoid (or at least think twice about using) in Math Papers:

- *This was given* (also: *as given*). If you're doing a proof, this is fine, but it's passive. If you're working on a specific situation that a specific person asked you to solve, you can be more direct. For instance, you could say *Using the information you provided.*)
- *Plugging in the numbers ...* Usually, you want to sound less mechanical than this – you want to sound like you're actively involved in finding a solution.

- *Using guess-and-check* (also: *we played around until we found something that worked*) In these sorts of situations, your readers need to know what you found and why it works, not all the unsuccessful attempts, *unless* for some reason you think that your failed attempts are somehow instructive.

0.5 Following the Checklist:

When you turn in your writing assignment, you should use a paper clip to attach the checklist to the front. You should feel free to use both the checklist and this booklet as a guide while you write, because you will be graded directly on the criteria outlined on the checklist. What follows here is a more detailed explanation of the criteria I use for grading your papers.

1. **Does this paper immediately (re)state the problem to be solved, with background and context? Follow the statement of the problem with your plan of attack? Immediately follow the plan of attack with a brief statement of the solution, in simple everyday terms?**

   - **Clearly (re)state the problem to be solved.**
     Do not assume that the reader knows what you're talking about. (The person you're writing to might be out on vacation, for example, or have a weak memory). Even if the person you're writing to is not memory-impaired, it serves as an introduction, gets your correspondent in the right frame of mind, and perhaps most importantly, makes clear what you interpreted the issues to be, so that if there was any misunderstanding, that's clear right away. You don't have to restate every detail, but you should explain enough so that someone who's never seen the assignment can read your paper and understand whats going on, without any further explanation from you. Outline the problem carefully, and be sure to include any data that was given to you.

     For example:

     > Dear John DeTect:
     > We've been hard at work solving your kidnapping problem, and you'll be happy to hear we've found the answer to your questions! ×

     as compared to

     > Dear Mr. DeTect:
     > You wrote to us asking for our help in determining when a kidnapping occurred and also whether one of the suspects can be eliminated based on some data you have. You wrote that on the day he went missing, eminent scientist Joe Bloe was last seen in his lab at 12 noon. At that time, he was in the midst of recording data from a top-secret experiment. His disappearance was discovered at 4:30 pm that same day. In the hopes that we can narrow down the time of his disappearance a bit from that 4 and a half hour window, you sent us the data he had recorded (leaving out, of course, all reference to what the experiment was about). Thinking it might help us, you also managed to obtain one more data sample after his disappearance was discovered. In addition to getting a better handle on when Dr. Bloe disappeared, you also asked us to investigate the alibi of prime suspect Jane Blain, the up-and-coming scientist in the lab next to Dr. Bloe's. You are apparently quite satisfied that her alibis from 12 pm to 3pm and from 3:45 pm to 4:30 pm are ironclad, but you question the somewhat circumstantial alibi she provided to you for the time period between 3 pm and 3:45 pm. She told you that during that time she was studying the behavior of a sample of cells and recording data, but nobody saw her – all you

> have is the data she recorded. You asked us how good this alibi is – you wanted to know whether her data was consistent, and whether she could have faked the data.
> Both Dr. Bloe's data and Dr. Blain's data are included on the next page.

Dr. Bloe's data

time (minutes)	quantity
$t = 0$	42
$t = 15$	90
$t = 30$	102
$t = 45$	96
$t = 60$	90
$t = 75$	102
$t = 90$	150
$t = 105$	252
4:45pm	3006

Dr. Blain's data

time	quantity
3:00	171
3:05	682
3:10	2735
3:15	10885
3:20	19035
3:25	27185
3:30	691809
3:35	2785289
3:40	11141351
3:45	44564497

✓

While there's no denying that the second paragraph is much longer, it contains all the relevant information that the authors were given. This gets the reader in the appropriate frame of mind for what is to come, as well as allowing the person who asked for help to make sure that his request was properly interpreted. The first paragraph is too abrupt to serve as a proper introduction. Notice also that the salutation is a bit awkward: we don't usually begin letters with "Dear John DeTect:".

- **Provide a paragraph that explains how the problem will be approached.**

  It's not polite to plunge into mathematics without first warning your reader. Carefully explain how mathematics is related to your problem, if it's a real-life problem. In particular, explain how you decided what broad mathematical concepts you'd be using – what's the connection between the real-life issues and the mathematical concepts? In other words, in simple everyday language give a preview of how you solved the problem. You may want to outline the steps you're going to take, giving some explanation of why you're taking that approach, but if you do this, work hard to avoid your explanation reading like a list. It's nice to refer back to this paragraph once you're deep in the thick of your calculations. Generally speaking, you should avoid using any mathematical words or phrases unless you can realistically expect your audience to be familiar with them. The main exception to this guideline is if you want to mention that a new concept is coming soon. In this case, you would be careful to introduce it in a non-threatening way, such as "we decided the behavior looked like something called a 'exponential function' ". Below are two examples of the beginnings of "plan of attack paragraphs". First, the not-so-great example:

  > In order to find out when Dr. Bloe was kidnapped, we first plotted his data. Next, we found a function that corresponded to the data, by guessing and checking. We did this because we wanted to know when that function would be 3006, because then we could find out what time went with $t = 105$, so we'd knew when he was kidnapped. Then we moved on to Dr. Blain's data, and tried to find a function that matched it. ×

  There are many problems with the above paragraph. First of all, it's boring. It reads somewhat like a list. Second of all, it doesn't explain *why* any of the steps were taken – the reader has to figure it out (or not) by reading all the way through to the conclusion. Instead, it just starts with the first step the authors took *after* having decided on an approach, and proceeds in more or less chronological order. Notice also that when the authors did try to explain why they proceeded as they did, it came dribbling out a little bit at a time: we did *this* because we wanted *that* so that we could do *the other thing*. Some other problems: the authors said they found the solution by guessing and checking. People who commission you to solve their problems do *not* want to think you found a solution by guessing. This does not inspire confidence. Instead of saying they guessed, the authors should have said something about using their vast background knowledge to determine a function that would model the data. Also, on a more subtle point, the authors said that they would be able to find out when the kidnapping occurred, when in fact the most they could hope for was to narrow down the range of time during which the kidnapping occurred.

The next plan of attack demonstrates some of these points:

> You presented us with two tasks: to narrow down the time of Dr. Bloe's kidnapping, and to assess the quality of Dr. Blain's alibi, by assessing her data. We decided to first determine whether we could better pinpoint the time of the kidnapping – after all, if the kidnapping occurred after 3:45 pm, then Dr. Blain's second ironclad alibi will render your second question moot.
>
> Dr. Bloe's data began at $t = 0$ and ended at $t = 105$. Assuming that no entries in Dr. Bloe's notebook have been faked, we can assume that he disappeared at some point *after* $t = 105$ – whenever that was. If we can figure out what time of day $t = 105$ corresponded to, then we might have a better idea of when he disappeared. Since the data that Dr. Bloe recorded has no reference to times of day, we would have had no hope of figuring this out without that last piece of data you obtained. We decided to try to find a function that would model the data; we could then use that to find out roughly at what time t the mysterious quantity Dr. Bloe was measuring would have reached 3006. Since we know that this t would correspond to the real time of 4:45 pm, we could then count backwards to find out roughly what time of day corresponded to $t = 105$. If the time we find is any time after noon, we will have narrowed the window during which the kidnapping occurred. In order to find a function that would model Dr. Bloe's data, we looked at a graph of his data and used our experience with a variety of different type of functions to determine what type of function it most resembled. From there, it was a matter of algebra and arithmetic to come up with a function that modeled the data.
>
> Assessing the quality of Dr. Blain's alibi involved much the same procedure. We wanted to know whether her data was consistent, and whether she could have faked it. We again looked at a graph of her data, and found a function that fit *most* of her data remarkably well. We then looked to see whether it would fit *all* of her data. ✓

- **Immediately follow the plan of attack with a brief statement of the solution, in simple everyday terms.**

  One major difference between mathematical writing and most other writing styles is that you do not save the excitement of the solution for the conclusion. You give a brief statement of the result(s) you found write after describing your approach. This hopefully has the effect of whetting their appetite to find out more about how you found it. Failing that, it at least tells them the answer early on. As with the paragraph describing your approach, describe your results in everyday language, and in the context of their question(s).

2. **Does this paper clearly, precisely and convincingly explain how the solution was found, to the intended audience?**

   Carefully explain the structure of your solution so that one idea flows into the next in a way that is easy to follow. Your goal is to convince your audience that you solved the problem correctly. While you dont need to give all the algebraic details (see Section 2), your reader shouldnt feel like he or she needs to do any of the algebra either, so give enough detail so that he or she can follow along. At every step, you should be sure you are convincing the reader that your results make sense. You explanation should be rooted in the context of the problem, rather than an abstract series of calculations. If you were initially asked about how long it takes a cup of coffee to cool, than repeatedly refer to the temperature of the coffee, the temperature of the kitchen, the rate the temperature changes etc, rather than t(x), T(x) and T'(x).

If your solution has several cases that are essentially the same, you do not need to explain each case in the same detail. After carefully explaining one case, you can refer to it when explaining the other solutions. For example, you could use phrases like "A similar analysis shows . . ." or "Following the same reasoning as above, we see that . . .". However, if you do this, **be certain that the same line of reasoning does hold!!!**

There is no need to explain false starts – ideas that you tried which led nowhere. Only include a false start if it was the *obvious* thing to try and you want to explain why you're pursuing something much more complicated, *or* if the false start was instructive in some way.

3. **Does this paper clearly state the ideas and assumptions behind the methods you used, their connection to the problem, and why you approached it this way?**

    - What physical assumptions do you have to make? For instance, did you have to assume no friction, or no air resistance? That something was lying on its side, or was far away from everything else? Were the measurements you were using exact, or approximations? What effect will that have on your results? Does your work represent a model that will roughly predict future behavior, or is it exact?
    - If you decide to use Newton's Law of Cooling, or to find a function that models the behavior of a population, or even to place a diagram that you were supplied with on a set of axes, explain how this is connected to your problem, and why you're doing it.
    - Don't pull formulas out of a hat, and don't use variables that you don't define. Either derive the formula for yourself in the paper, or explain exactly where you found it, so that other people can find it too.
    - Be sure that you are continually keeping your explanation in the context of the situation presented to you. Your goal is to be convincing. While abstract mathematics is beautiful to many people, you can not assume it is to your audience. You are solving a particular problem, so tailor your explanation to that situation. As often as possible, connect the mathematics that you are doing to the problem you were asked to solve.

4. **Does this paper solve the question(s) that was (were) originally asked? Consider all applicable situations? Contain correct mathematics? Provide a solution which makes sense in the real world?**

    - **Consider all applications, and solve the question(s) that was (were) originally asked.**

      Be sure that you have addressed **all** the questions in the project. Also consider whether there are separate cases you need to investigate.
    - **Contain correct mathematics**

      This should be self-explanatory.
    - **Provide a solution that makes sense in the real world.**

      Make sure that you have put the solutions into the context of the original problem, both in the original statement of the results and throughout the course of your discussion.

Beyond that, however, make sure that your solution makes *sense*. For instance, if you find that a mass of material contains *more* of radioactive material after time has passed than it did at the beginning, then you've clearly made a mistake somewhere since radioactive material decays over time.

5. **Connect the mathematics and the prose in a natural way that flows well?**

Your paper should flow smoothly. As with any paper, each paragraph should also begin and/or end with transition statements. These are particularly good places to refer back to your "plan of attack" paragraph. Begin the paragraph with a reminder – in everyday English – of what idea in your plan proceeds next naturally. Then transition into how you're going to accomplish this mathematically, and then begin to do it. Once again, I remind you to be sure that you are keeping your explanation in the context of the problem you were given.

Mathematics comes in paragraphs, clauses, and sentences just as English does, and your mathematics must make sense. Specifically, mathematical formulas are like clauses or sentences and so they need proper punctuation, too. Put periods at the end of a computation if the computation ends the sentence; use commas if it doesnt. An example follows.

> If Dr. Crannell's caffeine level varies proportionally with time, we see that
>
> $$C_t = kt,$$
>
> where C_t is her caffeine level t minutes after 7:35 am, and k is a constant of proportionality. We can solve to show that $k = 202$, and therefore her caffeine level by 11:02 am ($t = 207$) is
>
> $$\begin{aligned} C_{207} &= (202) \cdot (207) \\ &= 41,814. \end{aligned}$$
>
> In other words, she's seriously jazzed. ✓

Notice in the above example that you also need transitions between the two languages. Words and phrases like "we see that", "where", "is", etc come in handy here. See the section on good mathematical phrases for more suggestions.

A few more words on writing mathematics:

- Do **not** confuse mathematical symbols for English words. "=" and "#" are especially common examples of this mistake. The symbol "=" is only used in mathematical formulas and equations, not in sentences:

  > We let V = volume of a single mug and n = the # of mugs. Then the formula for the total amount of root beer is $R = nV$. ×
  >
  > We let V stand for the volume of a single mug and n represent the number of mugs. Then the formula for the total amount of root beer we can pour, R, is $R = nV$. ✓

- **Do**, however, use equal signs when you state formulas or equations, because mathematical sentences need subjects and verbs too.

  > Then the formula for the total amount R of root beer we can pour is nV. ×
  >
  > Then the formula for the total amount of root beer we can pour is $R = nV$. ✓

6. **Use language the audience can understand; avoid terminology, notation and variables, and explain them when they are unavoidable?**

- If you do not *have* to use mathematical terminology, don't. For example:

  When laying rectangular strips of carpet in an oval room, the process resembles that of taking a Riemann sum. Since you don't want gaps in your carpet, you always want the carpet to "overestimate" the floor, so on one side of the room you'd have a left sum, and on the other side you'd have a right sum.

  Since $R_n = \Delta x \sum_{i=1}^{n} f(a + i\Delta x)$, we just need to model the shape of the room with a function, use the width of the carpet strips as Δx and go from there. ×

  When laying rectangular strips of carpet in an oval room, you don't want any gaps, so you'll lay the carpet out until meeting the farthest portion of the oval that that particular strip can meet, and trim the rest. In order to find how much carpet you need, then, we just need to find out how long each strip needs to be. Since the area of a rectangle is just the product of the length and the width, we will then be able to add up all the strip-lengths and multiply by the width of the strips. In order to find the lengths of the strips, we'll need to find a function that models the shape of the room's floor. ✓

- If there's a quantity you use only a few times, see if you can get away with *not* assigning it a variable. As examples:

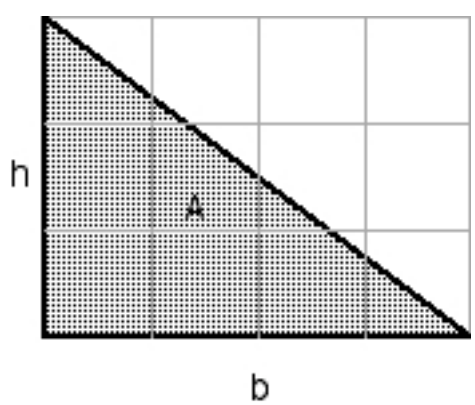

Figure 1: *Diagram of the Triangle* (Each square is 1" × 1".)

  We see that $A = \frac{1}{2}bh$, where A stands for the area of the triangle, b stands for the base of the triangle, and h stands for the height of the triangle, and so $A = \frac{1}{2} \cdot 3 \cdot 4 = 6$ square inches. ×

  We see that the area of the triangle will be half of the product of its height and base – that is, the area of the triangle is $\frac{1}{2} \cdot 3 \cdot 4 = 6$ square inches. ✓

  Elementary physics tells us that $v_t = g(t - t_0)$, where v_t is the velocity of the falling object at time t, g is gravity, and t_0 is the time at which the object is released. Therefore as t increases, so does v_t: i.e. as time increases, so does velocity. ×

  Elementary physics tells us that the velocity of a falling body is proportional to the amount of time it has already spent falling. Therefore, the longer it falls, the faster it goes. ✓

  I hope you agree that the second example in each pair is much easier to read.

- Even if you include a diagram that illustrates your variables (and if at all appropriate, you *should* do this), you should still explain in words what your variables represent.

- The more specific you are, the better. State the units of measurement. When you can use words like "of", "from", "above", etc, do so. For example,

  We get the equation $d = rt$, where d is the distance, r is the rate, and t is the time. ×

> We get the equation $d = rt$, where d is the distance from Sam's car to her home (in miles), r is the speed at which she's traveling (measured in miles per hour), and t is the number of hours she's been on the road. ✓

Avoid words like "position" (height above the ground? sitting down? political situation?) and time (5 o'clock? January? 3 minutes after the experiment started?).

- Variables that occur in the midst of text (prose) are italicized to tell them apart from regular letters.

7. **Include all relevant data (including any given to you)?**

   You can not assume that the person reading your essay is familiar with all the information, even that which they gave to you. As mentioned earlier, they may no longer have it available, but beyond that it makes your response flow and allows you to refer it. If you were given a table of data, then you must include that.

   If you calculated some data that you used to solve the problem, then of course, you must include that. In some cases, if the data is not central to the solution, you may include it in an appendix, but usually an explanation flows more smoothly if all the information is in front of the reader when it is being referred to.

8. **Does this paper include visual representations of the math (calculations, graphs, diagrams, charts, etc) where appropriate, and clearly introduce, label, explain, and refer to them?**

   - If your conclusions depended on a lot of calculations, you should include enough of those calculations so that your reader can follow what you're doing. If they are vital to the convincing the reader of the validity of your conclusions, then they should be included in the main body; if they are somewhat extraneous then you may include them in an appendix.

     A word on appendices: No appendix should stand on its own; an appendix should always be referred to by name within the main body of the text.

   - Put important or long formulas on a line of their own, and then center them; it makes them much easier to read:

     > The total number of infected cells in a honeycomb with n layers is $1+2+\cdots+n = n(n+1)/2$. Therefore there are $100(101)/2 = 5,050$ infected cells in a honeycomb with 100 layers. ×

     > The total number of infected cells in a honeycomb with n layers is
     >
     > $$1+2+\cdots+n = \frac{n(n+1)}{2}.$$
     >
     > Therefore there are $100(101)/2 = 5,050$ infected cells in a honeycomb with 100 layers. ✓

   - In math, even more than in literature, a picture is worth a thousand words, especially if it's a well-labeled picture. Whenever you find yourself trying to describe how something looks, include a diagram or picture. However, that picture needs to be in context.

     Include a reference to, and a brief description of, any picture in the body of your essay.

     Label all axes, with words, if you use a graph. Give diagrams a title describing what they represent. It should be clear from the picture what any variables in the diagram represent. The whole idea is to make everything as clear and self-explanatory as possible.

     Your paper will look much more formal if the pictures are drawn by computer (using Maple, or whatever drawing program rings your bell). However, you can spend a lot of time trying to get the pictures to look right, and then getting them to be included properly in your paper, and a well-drawn hand-generated picture is usually fine. It's up to you how you choose to balance your time.

9. **Does this paper give references and/or acknowledgement, when appropriate?**

    - While you are doing your best to write to a general audience, there will be situations where explaining all the details of a particular small part of your solution would be unnecessarily confusing. First, try to think if there's an alternative approach. If it seems as if you must include this small part, then you may want to refer the reader to a reference (often, but not always, your text).
    - Even if you have given a brief explanation of a background notion you're using, as it applies to the situation at hand – briefly referring to integration as area, for instance – you may want to give a further reference for if they want to pursue the concept in further detail.
    - Plagiarism is almost certainly the greatest sin in academia – some fiction writers make plagiarism a motive for murder. It's extremely important to acknowledge where your inspiration, your proofreading, and your support came from. In particular, you should refer to any book or article you look at as well as any computational or graphical software which helped you understand or solve the problem. You should also acknowledge any student you brainstormed with or who helped you (whether they're in this class or not) and any professor or TA you consult with (including me). The more specific you are the better. You may work these references and acknowledgements into the main body of the paper where they fit in naturally, or save them for the end.

10. **Does this paper appear legible, neat, and thoroughly proofread??**

    It may surprise you that people tend to lose many points on math papers due to grammatical and spelling mistakes. *Please* spell-check and proofread your paper for grammar mistakes. Mathematicians are not (generally) petty, but neither are we amused by sloppy or careless writing. Whether one is reading a discussion of the iconography that appears in a Renaissance painting or a discussion of the use of differential equations to determine a time of death, poor grammar and typos can severely interrupt the flow.

    Remember when proofreading to read the mathematics as sentences and clauses also.

    Ask a friend who is not in the class to read your paper, and then ask them to be honest about what they had trouble following.

    Please, please, please have every member of your group proofread the final draft of your paper before you give it to me.