Some Sample Problems for Midterm 1

Midterm 1 will cover Sections 4.1 through 4.7 and 5.1, as well as the relevant portions of 0.4 and 2.8.

Exams are more conceptual than homework - homework is for practicing concepts; exams are to check mastery of concepts.

You may bring one sheet of notes to the exam. This sheet should consist of no more than one side of an 8 1/2 x 11 sheet of paper. The notes should be handwritten by you (no photocopying tables or someone else's cheat sheet, for instance). Why? Because the main reason I allow a sheet of notes is because I think the process of creating one is valuable.

Because there are no classes between 12:30 and 1, you may begin the exam at 12:30pm. (You may not continue taking the exam beyond 1:55, however, no matter when you begin.)

You probably don't need me to remind you of this, but studying should consist not only of deciding what to put on your cheat sheet, re-reading the text, class notes, and returned problem sets, but most importantly, of actually **doing problems**. Re-do homework problems, or choose extra problems to do, or both. There is a review section at the end of each chapter with extra problems. Below, I have provided a few sample problems, but ...**the more problems you do, the better.**

In short: The following problems are intended as a supplement to your review; they are not intended to replace other review.

Another word of caution: You are responsible for all material covered in your reading, whether or not we covered it in class.

Before this exam, you should:

1. be able to differentiate quickly and accurately. This includes being able to use the product, quotient and chain rule, as well as of course remembering the derivatives of "elementary" (that is, "building block") functions such as trig functions, e^x and $\ln(x)$. If you need to, review the sections on the product, quotient, and chain rules in whatever book you have access to.

2. be able to *anti*differentiate quickly and accurately as well – see problems 5-30 in Section 4.1. *I am not including any straight antidifferentiation problems on this study guide.*

3. thoroughly understand the concept of the area function $\int_a^x f(t)\,dt$.

4. thoroughly understand both versions of the Fundamental Theorem of Calculus.

5. be able to find the derivatives of inverse trig functions in Section 2.8, problems 29-38.

6. understand where the derivatives of all the inverse trig functions came from, and be able to derive them–that is, be able to *derive* the derivative of $\arccos(x)$, $\arcsin(x)$, $\arctan(x)$, etc from scratch. (If you choose to do this by using the generic formula

for the derivative of an inverse function, then you should be able to first derive *that* formula, as we did not cover it in class.)

7. understand the ideas behind substitution (how it is related to the chain rule), as well as how to use it.
8. be able to compute most of the integrals in Section 4.6, problems 5-40 (they vary in difficulty, of course). **I am not including any of these on this study guide– master those in your text! Do as many as you need to.**
9. understand the definitions of the left, right, midpoint, and trapezoidal rules. Given an integral and a specific (small) number of subintervals, you should be able to draw the associated rough sketch, and use this sketch to write down any of these approximations without sigma notation.
10. understand sigma notation, how it works, what is a variable and what is not
11. be able to write left and right sums in sigma notation
12. understand how trapezoidal sums are related to left and right sums
13. understand how the shape of a curve can determine whether a specific approximation method over- or under-estimates the value of the integral
14. know the difference between the approximation, the actual error from the approximation, and an error bound for the approximation.
15. understand what an error bound is
16. understand what Theorem 7.1 tells you.
17. be able to find the area between two curves in either x or y - whichever makes the most sense in a particular situation.
18. You should be able to do all the problems below.

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1. Carefully explain why $\frac{d(\arctan(x))}{dx} = \frac{1}{1+x^2}$. That is, derive this differentiation formula.
2. Write each of the following in summation notation:
   (a) The sum of the square roots of the first 10 even integers
   (b) The square root of the sum of the first 10 even integers

   (There is no need to actually *find* these sums)
3. Differentiate $\sum_{j=1}^{10} \frac{1}{j}$

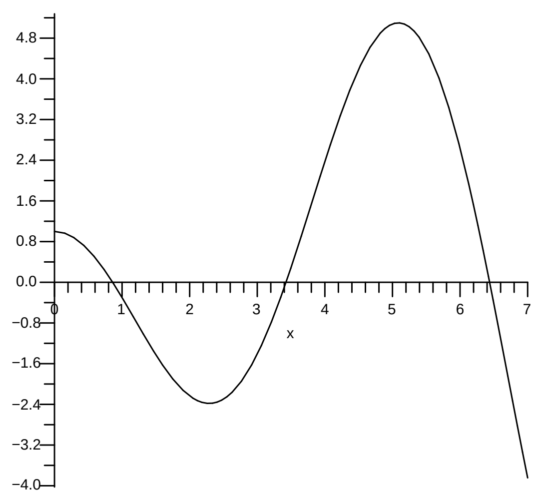
4. If $|f(x)| \leq 3$ for all x and $G(x) = \int_{-5}^{x} f(t)\,dt$, could $G(10) = -20$?

5. If $F(x) = \displaystyle\int_0^{x^2} \frac{1}{\sqrt{1-t^2}}\,dt$, find the equation of the line tangent to $y = F(x)$ at $x = \dfrac{1}{\sqrt{2}}$.

6. Is $f(0)\cdot 1 + f(2)\cdot 2 + f(4)\cdot 3 + f(6)\cdot 4$ a Riemann Sum approximation to $\int_0^{10} f(x)\,dx$? Justify your answer. [HINT: Draw a picture.]

7. Let $I = \displaystyle\int_3^8 f(x)\,dx$. Draw the graph of a function f where $0 < R_n < T_n < I < L_n$.

8. Let f be a function whose second derivative is shown below. Find n so that T_n is within 0.001 of $I = \displaystyle\int_0^{3.5} f(x)\,dx$



9. For each case, find a definite integral $\displaystyle\int_a^b f(x)\,dx$ for which $5\displaystyle\sum_{k=1}^{3} f(5k)$ is

   (a) a right Riemann sum with 3 equal subintervals
   (b) a left Riemann sum with 3 equal subintervals
   (c) a midpoint Riemann sum with 3 equal subintervals

   Clarification: you don't need to find f itself, just a and b.

10. Rewrite $\int_0^{\pi/4} \sec^2(x) f(\tan(x))\,dx$ using a substitution.

11. Sketch and find the area of the region bounded by $y = x^3$ and $y = 4x^2 - 4x$