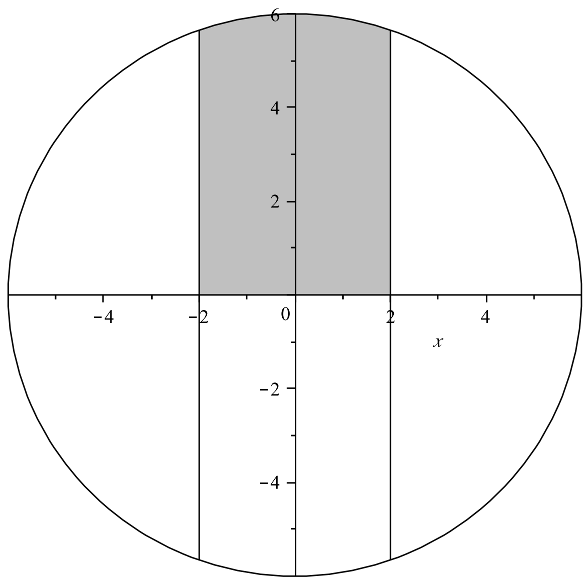
1. Suppose that a 12" diameter pizza is cut into "thirds" by making vertical cuts 2" on either side of the center (so that each "third" is 4" wide). How much bigger is the center piece than each of the other two pieces? You may use Maple.

   The pizza looks like this:



   Plan: To answer the question, we'd like to know $\frac{\text{area of center slice}}{\text{area of 1 side slice}}$. This ratio is the same as the one we get by focusing only on the top half. We can either use integrals to find both areas, or use an integral to find the area of the center slice, then subtract it from the entire area of the upper semi-circle and divide the result in half to find the area of one slide slice.

   In order to do this, we need to know the formula that produces the graph of the upper semi-circle.

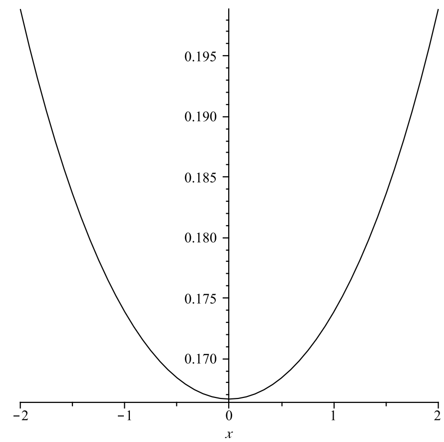
$$x^2 + y^2 = 36 \Rightarrow y = \pm\sqrt{36 - x^2}.$$

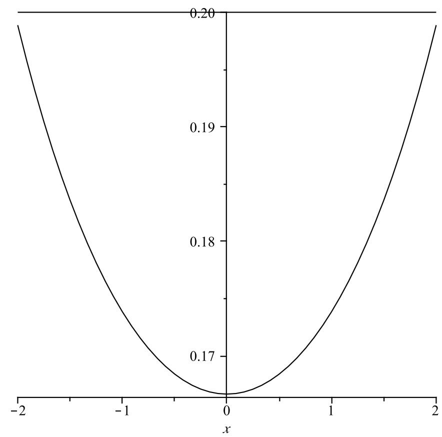
   Thus

$$\text{Area of top half of center slice} = \int_{-2}^{2} \sqrt{36 - x^2}\, dx.$$

   This is not an integral we can evaluate exactly, so we will use a midpoint sum within 0.001.

   Use Theorem 7.1 to find the number of subintervals. First I need to find K, so I need to look at the graph of the absolute value of the second derivative of $\sqrt{36 - x^2}$. Using Maple, I first graphed just this absolute value - see the lefthand graph below. Based on that, it looked as if 0.2 is always larger than the absolute value of the second derivative over $[-2, 2]$, but I graphed the horizontal line $y = .2$ just to be sure (shown on the right).





Based on these graphs, I can see that I can use $K = 0.2$.

$$\begin{aligned} \frac{0.2\big(2-(-2)\big)^3}{24n^2} &\leq 0.001 \\ \frac{12.8 * 1000}{24} &\leq n^2 \\ 533.33 &\leq n^2 \\ 23.09 &\leq n \end{aligned}$$

Thus M_{24} will approximate the area of the top half of the center slice to within 0.001. Using Maple, I find that M_{24} is 23.549, so

$$\text{area of top half of center slice} = 23.549 \pm 0.001.$$

Since the total area of the pizza is

$$18\pi \approx 56.549,$$

the area of the top half of each side slice is approximately

$$\frac{18\pi - 23.649}{2} \approx 16.500.$$

Thus the ratio of the area of the center slice to a side slice is approximately

$$\frac{23.549}{16.500} = 1.427,$$

so the center slice is nearly half again as big as either end slice.