Daily WeBWorK, Problem 1

1. Given $A_n = \dfrac{70}{7^n}$, determine:

   (a) whether $\displaystyle\sum_{n=1}^{\infty} (A_n)$ is convergent.

$$\sum_{n=1}^{\infty} (A_n) = \sum_{n=1}^{\infty} \frac{70}{7^n}$$ Looks like it may be a geometric series

- Need to be clear that it's of the form $a \displaystyle\sum_{n=\text{something}}^{\infty} r^n$
- The formula $\displaystyle\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$ only works if we start with $n = 0$.
- Method 1:

$$\sum_{n=0}^{\infty} \frac{70}{7^n} = 70 + \frac{70}{7} + \frac{70}{7^2} + \frac{70}{7^3} + \ldots = 70\left(1 + \frac{1}{7} + \frac{1}{7^2} + \frac{1}{7^3} + \ldots\right)$$

$$= 70\left(\sum_{n=0}^{\infty} \frac{1}{7^n}\right) = 70\left(\frac{1}{1-1/7}\right) = 70\left(\frac{7}{6}\right)$$

$$\sum_{n=1}^{\infty} \frac{70}{7^n} = \frac{70}{7} + \frac{70}{7^2} + \frac{70}{7^3} + \ldots = \sum_{n=0}^{\infty} \frac{70}{7^n} - 70 = 70\left(\frac{7}{6}\right) - 70$$

Daily WeBWorK, Problem 1 (continued)

1. Given $A_n = \dfrac{70}{7^n}$, determine:

   (a) whether $\sum_{n=1}^{\infty} (A_n)$ is convergent.

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- Need to be clear that it's of the form $a \sum_{n=\text{something}}^{\infty} r^n$
- The formula $\sum_{n=0}^{\infty} r^n = \dfrac{1}{1-r}$ only works if we start with $n = 0$.
- Method 2:

$$\begin{aligned}
\sum_{n=1}^{\infty} \frac{70}{7^n} &= \frac{70}{7} + \frac{70}{7^2} + \frac{70}{7^3} + \dots \\
&= \frac{70}{7}\left(1 + \frac{1}{7} + \frac{1}{7^2} + \frac{1}{7^3} + \dots\right) \\
&= \frac{70}{7} \sum_{n=0}^{\infty} \frac{1}{7^n} = \frac{70}{7}\left(\frac{1}{1-\frac{1}{7}}\right) = 10\left(\frac{7}{6}\right)
\end{aligned}$$

Daily WeBWorK, Problem 1 (continued)

1. Given $A_n = \dfrac{70}{7^n}$, determine:
   (b) whether $\{A_n\}$ is convergent.

$$\lim_{n\to\infty} A_n = \lim_{n\to\infty} \frac{70}{7^n} = 0$$

Thus the sequence of terms is convergent, and it converges to 0.

Daily WeBWorK, Problem 2

2. Given that $A_n = \dfrac{9n}{-2n+7}$, determine

   (a) whether the series $\sum_{n=1}^{\infty} A_n$ converges or diverges.

   We don't know how to deal with anything other than geometric series yet, but we do know a series converges if it's sequence of partial sums converges, so let's look at those. I'll just do a few:

$$S_3 = \sum_{n=1}^{3} \frac{9n}{-2n+7} = \frac{9}{-2+7} + \frac{18}{-4+7} + \frac{27}{-6+7} = \frac{174}{5} \approx 34.8$$

$$S_7 = \sum_{n=1}^{7} \frac{9n}{-2n+7} = \ldots = -36$$

$$S_{10} = \sum_{n=1}^{10} \frac{9n}{-2n+7} = -\frac{-8335}{143} \approx -58.3$$

   It appears that this goes up at first, but then starts going down, quite rapidly. Looks like it diverges to $-\infty$

Daily WeBWorK, Problem 2

2. Given that $A_n = \dfrac{9n}{-2n+7}$, determine

   (b) whether the sequence $\{A_n\}$ converges or diverges.

$$\lim_{n\to\infty} \frac{9n}{-2n+7} = \text{(using l'Hôpital's Rule)} - \frac{9}{2}$$

Recall: Goal

Find some techniques to help us determine whether a series is finite or is infinite.

Progress so far: If a series is of the form $a\sum_{k=0}^{\infty} r^k$, where a is any constant, then if $|r| < 1$, the series converges (to $\frac{a}{1-r}$), while if $|r| \geq 1$, the series diverges

But most series *aren't* geometric series, so we'd like to learn more.

Series We've Seen

Conv	$\sum_{k=0}^{\infty} \frac{1}{2^k}$	$\sum_{k=0}^{\infty} \frac{1}{\sqrt{2}^k}$	$\sum_{k=1}^{\infty} \frac{1}{k^2}$	$\sum_{k=0}^{\infty} \frac{3^k}{(-4)^k}$	$\sum_{k=0}^{\infty} \frac{4}{3^k}$	$\sum_{k=1}^{\infty} \frac{50}{5^n}$
	$\frac{1}{2^k} \to 0$	$\frac{1}{\sqrt{2}^k} \to 0$	$\frac{1}{k^2} \to 0$	$\frac{3}{(-4)^k} \to 0$	$\frac{4}{3^k} \to 0$	$\frac{50}{5^n} \to$
Div	$\sum_{k=0}^{\infty} 2^k$	$\sum_{k=0}^{\infty} 1$	$\sum_{k=0}^{\infty} \left(\frac{4}{3}\right)^k$	$\sum_{k=1}^{\infty} \frac{9n}{-2n+7}$		
	$2^k \to \infty$	$1 \to 1$	$\left(\frac{4}{3}\right)^k \to \infty$	$\frac{9n}{-2n+7} \to -\frac{9}{2}$		

Question: What characteristic of a series seems as if it would *guarantee* that the series will diverge?

Conjectures

Conjecture 1: If $\lim_{k\to\infty} a_k \neq 0$, then $\sum_{k=0}^{\infty} a_k$ can not converge.

Conjecture 2: If $\lim_{k\to\infty} a_k = 0$, then $\sum_{k=1}^{\infty} a_k$ converges.

Conjectures

Conjecture 1: If $\lim_{k\to\infty} a_k \neq 0$, then $\sum_{k=0}^{\infty} a_k$ can not converge.

Conjecture 2: If $\lim_{k\to\infty} a_k = 0$, then $\sum_{k=1}^{\infty} a_k$ converges.

One of these conjectures is true ...

Recall:

$$\sum_{k=0}^{\infty} a_k \stackrel{\text{def}}{=} \lim_{n\to\infty} \sum_{k=0}^{n} a_k$$

Or in other notation,

$$a_0 + a_1 + a_2 + a_3 + \ldots = \lim_{n\to\infty} \left(a_0 + a_1 + a_2 + \ldots + a_n \right)$$

We call $\sum_{k=0}^{n} a_k$ (otherwise known as $a_0 + a_1 + a_2 + \ldots + a_n$), the nth partial sum, and we denote it S_n.

2 sequences associated with the Harmonic Series

For the Harmonic series $\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \ldots$:

Sequence of Terms a_k	Sequence of Partial Sums S_n
$a_1 = \frac{1}{1} = 1$	$S_1 = \sum_{k=1}^{1} \frac{1}{k} = \frac{1}{1} = 1$
$a_2 = \frac{1}{2}$	$S_2 = \sum_{k=1}^{2} \frac{1}{k} = 1 + \frac{1}{2} = \frac{3}{2}$
$a_3 = \frac{1}{3}$	$S_3 = \sum_{k=1}^{3} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$
$\vdots$	$\vdots$
$\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots\}$	$\{1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \ldots\}$

Goal:

DarkBlueFind some techniques to help us determine whether a series converges or diverges

Methods we have so far:

Given a series $\sum_{k=M}^{\infty} a_k$,

- **Is it a geometric series?** If so, we can determine whether or not it converges, and if so, exactly what it converges to.
- **Divergence Test:** If $\lim_{k\to\infty} a_k \neq 0$, the series diverges. If $\lim_{k\to\infty} a_k = 0$, inconclusive.

But so many series aren't geometric *and* have $\lim_{k\to\infty} a_k = 0$. How do we deal with *those* series?

For instance, does $\sum_{k=1}^{\infty} \frac{1}{k^3}$ diverge or converge?