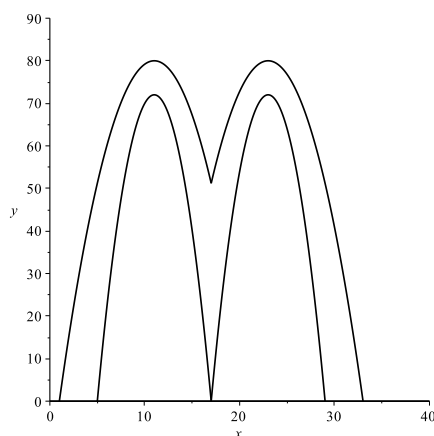


Group Problem Set 1

1. If $F(x) = \int_0^{x^2} \frac{1}{\sqrt{1-t^2}} dt$, find the equation of the line tangent to $y = F(x)$ at $x = \frac{1}{\sqrt{2}}$.
2. The Lovelytown Zoning Commission has before it a proposal from MacDougal's Burgers and Wings to erect a giant sign next to the freeway at a site just inside the borders of Lovelytown. Local ordinances require that any sign within the borders of Lovelytown must have frontal area of 800 square feet or less. The sign was originally rejected on the grounds that its base is 32 feet wide and its height is 80 feet, leading to a rectangular area of well in excess of 2000 square feet. MacDougal's has appealed on the grounds that their sign is *not* a rectangle, but a stylized "M", and as such its area is considerably smaller than the Zoning Commission had assumed.

The sign, and the functions which give the boundaries of the stylized "M" are shown below. Calculate the exact frontal area of the "M" sign to determine whether MacDougal's sign meets the requirement of being under 800 square feet.

Shape of the MacDougal's Billboard



$$B_1(x) = 80 - \frac{4}{5}(x - 11)^2$$

$$B_2(x) = 80 - \frac{4}{5}(x - 23)^2$$

$$B_3(x) = 72 - 2(x - 11)^2$$

$$B_4(x) = 72 - 2(x - 23)^2$$

3. If $|f(x)| \leq 3$ for all x and $G(x) = \int_{-5}^x f(t) dt$, could $G(10) = -20$?
4. Carefully explain why $\frac{d(\arctan(x))}{dx} = \frac{1}{1+x^2}$. That is, derive this differentiation formula. Do not use any formulas for the generalized derivative of an inverse function - follow the technique used in class.