Determine whether $\sum_{j=2}^{\infty} \frac{1}{j(\ln(j))^5}$ converges or diverges. If the series converges, find a number N such that the partial sum S_N approximates the sum of the series within .001. If the series diverges, find a number N such that $S_N \geq 1000$.

- **Determining convergence or divergence:**
  - **Try the the nth term test:** I can immediately see that as $j \to \infty$, $a_j \to 0$, so the nth term test is inconclusive. Nonetheless, since it's usually quick to check, I should always at least give it some thought first.

    What test should I try next? When you're first starting with these analyses, you're not going to really know, so just pick one and try it. If it gets you somewhere reasonably quickly, great. If you find yourself hitting a wall (whether it's a comparison that isn't helpful *or* that you can't find a comparison *or* that you can't integrate your term), switch to a different test. If that also goes nowhere, you can always go back.

    Here's my thought process: I look at the term, and I see a $\ln(j)$ in the denominator, along with a j. I've learned from experience that this is a combination I can integrate, so I decide to try the integral test.
  - **Try the integral test:**
    - **Check that the integral test applies:**
      To do this, I need to verify that $\frac{1}{x(ln(x))^5}$ is continuous, positive and decreasing on $[2, \infty)$.
      1. On $[2, \infty)$, x and $\ln(x)$ both increasing $\Rightarrow x(\ln(x))^5$ increasing $\Rightarrow \frac{1}{x(\ln(x))^5}$ decreasing.
      2. $\frac{1}{x(ln(x))^5}$ is only undefined at $x = 0$ and $x = 1$, which aren't in the interval $[2, \infty)$, so our function is continuous.
      3. x is positive on $[2, \infty)$, and so is $\ln(x)$, so our function is positive.

Therefore the integral test applies.

* **Determine the convergence of** $\int_2^\infty \frac{1}{x(\ln(x))^5}\,dx$**.**

Now that I know the integral test applies, I know that if $\int_2^\infty \frac{1}{x(\ln(x))^5}\,dx$ converges, the series $\sum_{j=2}^\infty \frac{1}{j(\ln(j))^5}$ will also converge, and conversely if $\int_2^\infty \frac{1}{x(\ln(x))^5}\,dx$ diverges, the series will also diverge.

Let $u = \ln(x)$. Then $du = \frac{1}{x}\,dx$, and when $x = 2$, $u = \ln(2)$, and as $x \to \infty$, $u \to \infty$ also. Therefore,

$$\int_2^\infty \frac{1}{x(\ln(x))^5}\,dx = \int_{u=\ln(2)}^\infty \frac{1}{u^5}\,du,$$

which we know converges by the p-test (since $p = 5$ is greater than 1).

Since the integral converges, we know from the integral test that the series converges as well.

- **Fnd an N so that S_N approximates the sum of the series within .001.**

Since $S = S_N + R_N$, we know that $S - S_N = R_N$. That tells us that in order to find an N so that S_N is within .001 of S, all we have to do is find N so that $R_N \leq .001$.

Part of the integral test tells us that

$$R_N \leq \int_N^\infty a(x)\,dx,$$

so if I find N so that

$$\int_N^\infty \frac{1}{x(\ln(x))^5}\,dx \leq .001,$$

I'll be done.

Using the same substitution as I did when showing convergence, and that when $x = N$, u will be $\ln(N)$, I do the following calculations:

$$\begin{aligned}
\int_N^\infty \frac{1}{x(\ln(x))^5}\,dx &\leq .001 \\
\lim_{t\to\infty}\int_{\ln(N)}^t \frac{1}{u^5}\,du &\leq \frac{1}{1000} \\
\lim_{t\to\infty}\left(-\frac{1}{4t^4}+\frac{1}{4(\ln(N))^4}\right) &\leq \frac{1}{1000} \\
4(\ln(N))^4 &\geq 1000 \\
(\ln(N))^4 &\geq 250 \\
\ln(N) &\geq 3.98 \\
N &\geq e^{3.98} \\
N &\geq 53.52 \\
N &= 54
\end{aligned}$$

Therefore, $\displaystyle\sum_{j=2}^{54}\frac{1}{j(\ln(j))^5}$ is within .001 of $\displaystyle\sum_{j=2}^{\infty}\frac{1}{j(\ln(j))^5}$