Alternating Series Test:

Consider the alternating series

$$c_1 - c_2 + c_3 - \cdots = \sum_{k=1}^{\infty} (-1)^{k+1} c_k,$$

where $c_k \geq 0$.

If $\lim_{k \to \infty} c_k = 0$, then the series converges and its limit lies between any two consecutive partial sums. That is, if the series converges to S, then S lies between S_n and S_{n+1} for any n.

In particular, the error in using S_n to approximate S is bounded as follows:

$$|S - S_n| \leq c_{n+1}.$$

The following series converge (one absolutely, the other conditionally). In each case, calculate S_{1000} using Maple, and determine how close this approximates the value of the series.

1. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{k^2+1}$

2. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^5}{n^6+17}$