

A **power series** is any series of the form

$$a_0 + a_1x + a_2x^2 + \dots + a_nx^n + \dots = \sum_{k=0}^{\infty} a_kx^k,$$

or more generally

$$\begin{aligned} a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + \dots + a_n(x - x_0)^n + \dots \\ = \sum_{k=0}^{\infty} a_k(x - x_0)^k. \end{aligned}$$

A power series will converge over an interval centered at the base-point  $x_0$ . This interval is called the **interval of convergence**.

That interval may consist only of the base point, or it may be larger. Some power series converge for all  $x$ .

To find the interval of convergence of a power series

$\sum_{k=0}^{\infty} a_k(x - x_0)^k$ , use the ratio test on

$$\sum_{k=0}^{\infty} |a_k(x - x_0)^k| = \sum_{k=0}^{\infty} |a_k| \cdot |x - x_0|^k.$$

1. Where  $L < 1$ , the power series will converge absolutely.
2. Where  $L > 1$ , the power series will diverge.
3. Where  $L = 1$  (which will be the two endpoints of your interval of convergence), you need to test further.